2018 Annual American Control Conference (ACC)
June 27–29, 2018. Wisconsin Center, Milwaukee, USA

Jump Linear Quadratic Control for Energy Management of a Nanogrid*

Nathaniel Tucker, *Student Member IEEE*, and Maryam Khanbaghi, *Senior Member IEEE*

Abstract— **Energy security and the aging grid infrastructure in the United States are two important subjects at the crossroads between politics and technology. One proposed solution to these two issues revolves around the concept of an islanded microgrid. Microgrids are becoming increasingly popular, combining multiple energy generation and storage systems to diversify and secure their private energy portfolios. The work presented in this paper is the application of jump linear quadratic (JLQ) control to the energy management of an islanded microgrid operating with a solar generation, fuel cell, and battery energy storage system. A continuous finite state Markov chain is used to model the intermittent generation of the solar array. The microgrid's energy management model is formulated, stochastic optimal feedback control is obtained, and results from an example illustrate the validity of the proposed approach.**

I. Introduction

With prices of consumer photovoltaic (PV) generation systems decreasing, the number of small-scale electricity generating households is growing steadily. Specifically, in California, where the weather is advantageous for solar generation, average citizens are installing PV generation systems at an impressive rate [1]. While this is a positive development that will help migrate energy production away from centralized fossil fuel plants by increasing the customers' control of their own electricity generation, many current household generation systems are not able to take full advantage of the electricity they generate. Households solely equipped with PV systems can only generate electricity when there is sun (daytime) and are forced to purchase electricity from the grid otherwise. Additionally, excess generated energy has to be sent back into the grid, instead of being stored for later use, which brings another layer of complexity to managing grid stability.

To circumvent this set of problems, several energy storage systems have emerged within the market that give households the ability to store self-generated energy for later use. These battery energy storage systems (BESS) facilitate several beneficial functions such as solar self-consumption, time of use load shifting, backup power, and off-the-grid use. With the increasingly common partnership of PV generation and battery energy storage systems, a household with both can be treated as its own microgrid system, capable of self- generation, storage, and only using a connection to the existing utility network when absolutely necessary. The major component of microgrid energy management systems is advanced control strategies. Hence, intense research is being conducted on optimal energy management of microgrids. In this paper, we name a few recent publications pertinent to our work. Wu *et al.* [2] focuses on stochastic energy management of a home with plug-in electric vehicle energy storage and PV. This work seeks to minimize consumer energy charges. Similarly, Beloni *et al.* [3] focuses on multi-load systems utilizing shared resources with the goal of lowering overall energy cost. Furthermore, Dong *et al.* [4] considers an optimal stochastic control for home energy systems with solar and energy storage where the demand is subject to Brownian motions. In [5], the microgrid energy management problem is formulated as a two-stage stochastic programming considering uncertainty. In a more recent work [6], a Markov jump process is used to model the stochastic changes of distributed energy storage systems.

Proposed control strategies assume the microgrid has access to the public distribution grid to satisfy load demand when necessary. The challenge we address in this work would be to eliminate this connection which will help to reduce the burden on the power grid. By eliminating this connection, we need to utilize a proxy distributed generation source. This will add an additional control input to the system. The control strategy proposed focuses on small off-the-grid systems; more specifically, residential homes. We will call these single home microgrids, *nanogrids* (nGrids). Nanogrids are a new concept [7] and are defined as micorgrids under 5 kW. In this work, the particularity of nGrids is that they have no connection to the electric grid. The assumption is that every nGrid will have a Renewable Energy Source (RES) such as rooftop solar panels, storage, and a sustainable clean energy generator such as a fuel cell. We made this decision not out of convenience, but to further support the reduction of fossil fuel consumption. This assumption also brings a few advantages. The control strategy:

- does not need to handle multiple loads and control competition for a shared resource,
- relinquishes power generation and distribution responsibilities from the public grid,
- provides the consumer complete control.

Solar energy used as a RES is a natural choice for residential homes. The price of solar panels is continuously decreasing, resulting in their increased proliferation. One of the characteristic features of solar energy generation is the impossibility of predicting the exact time of occurrence, the duration, and the quality of the energy provided. Hence, it is not optimum to design a control strategy using a deterministic model. In this work, we make the assumption,

* Research supported by SCU School of Engineering Internal Grant.
Nathaniel Tucker was with the Electrical Engineering Department, Santa Clara University, CA 95053. He is now with the Electrical Engineering Department, UC Santa Barbara, Santa Barbara, CA 93106 (Nathaniel_tucker@umail.ucsb.edu).
Maryam Khanbaghi is with the Electrical Engineering Department, Santa Clara University, CA 95053, USA, (corresponding author phone: 408 554 6816; mkhanbaghi@scu.edu).

with backing of real data, that solar energy delivered could be modeled as a continuous Markov chain with three states. Consequently, it has become apparent to us that an appropriate framework for capturing and analyzing energy management problems of a nGrid is to utilize random jump dynamic system theory [8]. In the present work, we propose a control strategy for a nGrid energy management system where the average household can operate autonomously from the main grid with current technologies available in the market. Furthermore, contrary to other proposed control strategies [2-6], the one designed here is in continuous time where analytical solutions may be obtained.

This paper is organized as follows. In section II, a system architecture as well as specifics for each component are described. In Section III, jump linear quadratic control is introduced and applied to the proposed nGrid. In section IV, an example is presented with simulated results showing the effectiveness of the proposed strategy. In the final section, conclusions and directions for future research are presented.

II. System Description

A. Nanogrid Architecture

The microgrid system we are proposing is represented in Figure 1. The system is equipped with a photovoltaic array for generation during the daytime, a hydrogen fuel cell for baseline energy generation, and a battery energy storage system acting as a buffer to smooth intermittent generation and to optimize the energy generation/usage balance at night. Note that there is no connection to the standard utility power distribution network. This is an islanded microgrid that must generate energy to meet the load's demand. As mentioned in the introduction we call this type of microgrid a *nGrid*.

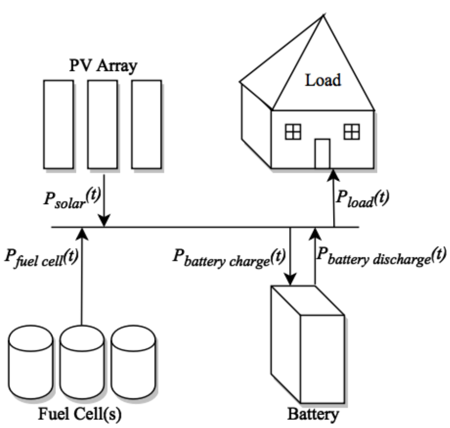

Fig. 1. nGrid architecture with battery, fuel cell, solar, and load.

The primary goal of the system presented here is to manage the energy demand of a single home without being connected to the grid. This is done by balancing the power generated by the PV array, fuel cell, and battery with the power required by the load $P_{load}(t)$ as:

$$P_{load}(t) = P_{solar}(t) + P_{FC}(t) + P_{batt.}(t). \quad (1)$$

$P_{load}(t)$ follows a preassigned trajectory described in the next section. Power generated by the PV array, $P_{solar}(t)$, follows a stochastic process that varies due to cloud coverage as well as time of day. The fuel cell's generated power, $P_{FC}(t)$, is limited to constant values depending on the generation capabilities of the specified fuel cell. Lastly, the battery $P_{batt.}(t)$ can inject power into the system, as well as absorb power when needed. In this architecture, $P_{load}(t)$ is the system set point, $P_{solar}(t)$ is an uncontrollable stochastic input modeled by Markov chain, $P_{batt.}(t)$ and $P_{FC}(t)$ are control inputs.

The PV array should be sized in a way that is proportional to the daytime load of the house, allowing full load satisfaction during sunny weather with the excess energy being sent to the battery. Likewise, it is desired for the fuel cell to act as a baseline generator and produce a constant amount of energy without the need to cycle between on and off states frequently. As stated earlier, the battery will act as a buffer which will accommodate use of stored solar energy when the PV array is not producing energy. The load is modeled after an average house in the U.S. with additive noise for slight fluctuations in energy demand. The following assumptions are made throughout this study:

- The system has full information on the state of the system including instantaneous power flow, maximum and minimum generation levels, as well as maximum and minimum battery levels. This would be implemented with a battery State of Charge (SoC) tracker combined with voltage, current, or phasor monitoring devices.
- Reactive power is not taken into account. It is assumed that the controllers/inverters on the PV array, battery, and fuel cell regulate voltage and the phase angle. Only active power is considered.
- The operation of the fuel cell is simplified. The source of hydrogen and oxygen fuel is not specified and no energy is used for electrolysis, compression, or generation.

B. Load Model

The load in the nGrid is modeled after an average house in the United States. The load consumption model comes from a National Renewable Energy Laboratory (NREL) System Advisor Model (SAM) dataset available online [9]. The goal of the nGrid energy management controller is to match this load through solar generation, fuel cell generation, and battery discharge.

C. Solar Generation Model

Solar power is becoming a highly desirable form of generation due to its passive generation profile and decreasing panel costs. However, solar generation possesses an intrinsic shortcoming which stems from its intermittent nature. PV generation requires sunny weather for max efficiency, and generation is severely obstructed by cloud coverage. To model this stochastic generation, a continuous Markov chain was used to represent cloud coverage patterns similar to the approach taken in [10]. As shown in Figure 2, cloud coverage falls into one of three categories during the day: sunny, cloudy, and overcast. We decided on only three states during the day for simplicity. Obviously, at night, there is no solar generation. Hence, the power generated by the PV array $P_{solar}(t)$ is presented by Eq. (2), where $P_s(r(t))$ takes constant power values that are related to the number of panels, efficiency, and load requirements. There is a deterministic switch to $P_{solar}^{night}(t) = 0$ during night.

$$P_{solar}(t) = \begin{cases} P_{solar}^{day}(t) = P_s(r(t)) = \begin{cases} r(t) = 1 & \text{(sunny)} \\ r(t) = 2 & \text{(cloudy)} \\ r(t) = 3 & \text{(overcast)} \end{cases}, \\ P_{solar}^{night}(t) = 0 \end{cases} \quad (2)$$

The term, $r(t)$ denotes the mode of the Markov chain corresponding to the cloud coverage during the day.

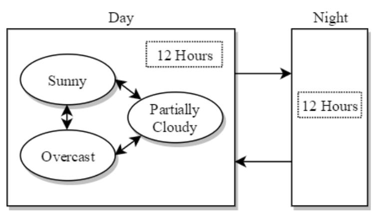

Fig. 2. Weather diagram with a Markov chain representing cloud coverage during daytime.

Therefore, $r(t)$ evolves according to a continuous time Markov chain taking values in a finite set $S = \{1,2,3\}$, with transition probability matrix $P = \{p_{ij}\}$ given by:

$$p_{ij} = Prob(r(t+\Delta) = j | r(t) = i) = \pi_{ij} + o(\Delta) \quad \text{if } i \neq j$$
$$= 1 + \pi_{ii}\Delta + o(\Delta) \quad \text{if } i = j$$

with $\lim_{\Delta \to 0} \frac{0(\Delta)}{\Delta} = 0$. Where $\Delta > 0, \pi_{ij}$ is the (non-negative) transition rate from i to j, $i \neq j$ and

$$\pi_i \triangleq -\pi_{ii} \triangleq \sum_{j=1, j\neq i}^{3} \pi_{ij} \quad (3)$$

with the matrix of the transition rates [8]:

$$\Pi = (\pi_{ij});\ i,j = 1,2,3.$$

Data for the cloud coverage transition probability matrix was gathered from San Jose International Airport's routine meteorological weather reports [10], also known as METAR. METAR data is collected at all major airports and government buildings in the United States and specifies temperature, precipitation, as well as cloud coverage. For our purposes, we were able to quantize the hourly cloud coverage from 2016 data into three levels: sunny, cloudy, and overcast. The Markov jump transition rate matrix calculated from the METAR data is below:

$$\Pi = \begin{bmatrix} -0.197 & 0.164 & 0.033 \\ 0.085 & -0.179 & 0.094 \\ 0.010 & 0.138 & -0.148 \end{bmatrix} \quad (4)$$

This is an adequate approximation. It is obvious that the probability to move from sunny to cloudy is higher than from sunny to rainy and vice versa. Furthermore, 2016 was a relatively rainy year for Silicon Valley, hence the magnitudes of the main diagonal are larger compared to the off-diagonal elements in the transition matrix. The cloud coverage system has a strong tendency to remain in one state for long periods of time.

D. Battery Energy Storage

To supplement the intermittent generation from the PV array, a battery energy storage system is employed alongside a baseline fuel cell generator. The fuel cell is used to provide energy to satisfy a portion of the load that the solar and battery cannot meet. The battery essentially acts as a buffer, storing excess solar and fuel cell energy for later use. The effectiveness of this fuel cell and battery combination has been demonstrated in [12]. The important specifications of the battery include energy capacity (more specifically SoC), charge/discharge powers, life cycle, as well as safe operating temperatures. SoC of a battery is its available capacity expressed as a percentage of its rated capacity. As it is not desired to deplete or overcharge the battery, the SoC of the battery should be kept within proper limits. Usually, SoC cannot be measured directly, but it can be estimated from direct measurement of voltage and current. Here, we make the assumption that energy stored in the battery $E_b(t)$ and the charge power $P_b(t)$ can be used to provide equivalent information on SoC. The battery has energy limits, $\underline{E}_b \leq E_b(t) \leq \overline{E}_b$, alongside charging and discharging limits, $0 \leq P_{batt}^{ch.}(t) \leq \overline{P}_b^{ch.}$, $0 \leq P_{batt}^{dis.}(t) \leq \overline{P}_b^{dis.}$, where $\overline{P}_b^{ch.}$ and $\overline{P}_b^{dis.}$ are the maximal allowable charge and discharge values, respectively, for the specified battery. For normal battery operation, it is common to set $\underline{E}_b \geq 10\%$ and $\overline{E}_b \leq 90\%$ of the maximum energy that can be stored to increase the lifespan of the battery.

E. Fuel Cell

Fuel cells are effective generators for long periods of time, from several minutes up to several months [14]. Because of this, it seems fuel cells are good candidates for energy generation for the nGrid. Similar to the battery, the fuel cell has generation limits $0 \leq P_{FC}(t) \leq \bar{P}_{FC}$. In our model, we make the assumption that the total amount of energy the fuel cell is capable of supplying over time is not restricted, which means that there is an essentially limitless supply of hydrogen and oxygen to power the chemical reaction. An electrolyzer and constant water supply could easily support this assumption. Adding the electrolyzer to the nGrid load as well as active hydrogen/oxygen tank monitoring will be considered in further work.

III. Mathematical Modeling and Jump Linear Quadratic (JLQ) Control Formulation

By analyzing and classifying the hourly weather condition, we suggest the solar generation process using roof- top solar panels could be seen as a continuous Markov chain with three states: a sunny state, a cloudy state, and an overcast state. Markov chain hypothesis mentioned in the introduction is central to our work. This hypothesis makes the resulting model more tractable mathematically; moreover, an important body of theory and applications exists for the control of jump linear Markov models [8,10,13,15]. In the following, we use the resulting Markov chain model of the solar generation process as a basis for the design of adequate control policies. Therefore, we first develop a simplified model of power balance dynamics with random solar input disturbances. Subsequently, a quadratic optimal control problem is formulated and the model is transformed so that the existing theory [8] can be applied.

A. Simplified Mathematical Model

Since the rate of change of energy is power, for this preliminary simplified analysis of the energy management of an nGrid, the power balance equations may be written as follows:

$$\frac{dE_b(t)}{dt} = \alpha E_b(t) + (1 - a_i)P_{solar}(t) - b_i P_{batt}^{dis.}(t) + (1 - c_i)P_{FC}(t) \quad (5)$$

$$\frac{dE_l(t)}{dt} = \beta E_l(t) + a_i P_{solar}(t) + b_i P_{batt}^{dis.}(t) + c_i P_{FC}(t) \quad (6)$$

The terms $E_b(t)$ and $E_l(t)$ correspond to the energy stored in the battery and the energy required by the load, respectively. We showed that in II.C, $P_{solar}(t)$ is reasonably modeled as a continuous Markov chain with 3 states and they are piecewise constant in each mode. Thus, the global system may be described as a continuous linear time invariant system with Markovian jumps and a hybrid (continuous-discrete) state space $[x_1\ x_2\ r]^T$. The scalar coefficients α and β take the appropriate units of $(\text{hour})^{-1}$ and are set equal to -1 for this analysis which ensures balanced power at steady state. If we define the state variables, output variables, control variables and stochastic input variables as

$$x(t) = \begin{bmatrix} E_b(t) \\ E_l(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} E_b(t) \\ E_l(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} P_{batt}^{dis}(t) \\ P_{FC}(t) \end{bmatrix},$$

$$P_{solar}(t) = P_s(r(t))$$

the state-space representation becomes

$$\begin{cases} \dot{x}(t) = Ax(t) + B(r(t))u(t) + P_s(r(t)) \\ y(t) = Cx(t) \end{cases} \quad (7)$$

where $x(t) \in \mathbb{R}^2$ is the continuous portion of the state and $r(t)$ is the discrete portion of the state and evolves according to a continuous time Markov chain, with

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad B(r(t)) = \begin{bmatrix} -b_i & 1 - c_i \\ b_i & c_i \end{bmatrix} = B_i,$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad P_s(r(t)) = \begin{bmatrix} 1 - a_i \\ a_i \end{bmatrix} P_i$$

for $r(t) = i$. $P_s(r(t))$ is power generated by the PV and assumed constant in each mode. Based on the nGrid components values a_i, b_i, c_i and P_i are chosen for each mode of operation. P_i being the power generated by solar panels at each mode i.

B. Jump Linear Systems Overview

The following jump linear optimal control overview is based on [8] and [14]. The state-space representation of a jump linear system has the form

$$\dot{x}(t) = A(r(t))x(t) + B(r(t))u(t); \quad x(t_o) = x_o \quad (8)$$

where $x(t) \in \mathbb{R}^2$ and $u(t) \in \mathbb{R}^2$ represent the plant state and input vector, respectively. $A(r(t))$ and $B(r(t))$ are 2×2 and 2×2 matrices, respectively, where $r(t)$ denotes the current system mode determined by a finite state Markov jump process. When the system is operating in the i^{th} mode, the corresponding system matrices $[A(r(t)),\ B(r(t))]\,|_{r(t)=i}$ will be denoted $[A_i, B_i]$. When designing an optimal controller for such a system, one aims to minimize the quadratic cost function,

$$J(u, t_0, r(t_0), x_0) = \mathbf{E}\left\{\frac{1}{2}\int_{t_0}^{t_f}\left(x^T(t)Q(r(t))x(t) + u^T(t)R(r(t))u(t)\right)dt \mid t_0, r(t_0), x_0\right\}, \quad (9)$$

t_0 being the initial time, t_f the final time and E{.} indicating expected value. The symmetric weighting matrices, $Q(r(t))$ and $R(r(t))$, are mode dependent and are used to tune the system response to fit desired characteristics. In the following, they will be denoted as $[Q_i, R_i]$ with $Q_i \geq 0$ (positive semi − definite) and $R_i > 0$ (positive definite) when the system is operating in its i^{th} mode. For the finite horizon problem with performance measure given by (9), the optimal regulator is given as a time varying feedback law:

$$u^*(t) = -R_i^{-1}B_i^T K_i(t)x(t) \quad \text{for} \quad r(t) = i \quad (10)$$

where matrices $K_i(t)$ $(i = 1,2,3)$ satisfy the set of coupled differential matrix Riccati equation:

$$\dot{K}_i(t) = -A_i^T K_i(t) - K_i(t)A_i - Q_i + K_i(t)S_iK_i(t) - \sum_{j=1}^{N}\pi_{ij}K_j(t) \quad (11)$$

with $S_i = B_i R_i^{-1} B_i^T$ and $K_i(t_f) = 0$.

In the following, we will be interested in steady state values of $K_i(t)$. Under stochastic controllability and observability conditions [13], the Riccati gains for the infinite horizon problem will converge to the unique positive definite solutions of the following set of coupled algebraic Riccati equations:

$$A_i^T K_i^\infty + K_i^\infty A_i + Q_i - K_i^\infty S_i K_i^\infty + \sum_{j=1}^{N}\pi_{ij}K_j^\infty = 0 \quad (12)$$

In case of the infinite horizon problem, the regulator minimizes the following performance measure:

$$J = \overline{lim}_{t_f\to\infty}\frac{1}{t_f}E\{\int_{t_0}^{t_f}(x^T(t)Q_ix(t) + u^T(t)R_iu(t))dt\}, \quad (13)$$

the resulting control law becomes:

$$u^*(t) = -R_i^{-1}B_i^T K_i{}^\infty x(t) \quad \text{for} \quad r(t) = i \quad (14)$$

The solution of Eq. (12) is obtained using the numerical algorithm presented in [14].

C. JLQ Control Applied to nGrid

Our Objective is to find a control law that will optimize the power required to meet the household energy demand. We assume that cost of energy is the same for all sources. In order to complete our model formulation as a standard framework proposed in [15], we do away with the piecewise constant vector $P_s(r(t))$ in Eq. (7) by means of the following change of variables:

$$\tilde{x}_1(t) = x_1(t) + (1 - a_i)P_i/\alpha \quad (15)$$

$$\tilde{x}_2(t) = x_2(t) + a_iP_i/\beta$$

This change of variable creates discontinuities of state trajectories at jump times. These discontinuities are due to constant solar power that differ from mode to mode. Thus, the resulting system model given by:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B(r(t))\tilde{u}(t) \quad (16)$$

at continuity points of $r(t)$ and by Eq. (17) at jump instants.

$$\tilde{x}(t) = \tilde{x}(t^-) + q_{ij} \quad (17)$$

with $q_{ij} = \begin{pmatrix} (1-a_i)P_i - (1-a_j)P_j \\ a_iP_i - a_jP_j \end{pmatrix}$

$\tilde{x}(t^-)$ is the limit from the left of $\tilde{x}(\tau)$ as τ goes to t and q_{ij} is the change due to constant perturbations that differ from mode i to mode j.

For the infinite horizon problem with performance measure given by Eq. (13), the optimal regulator is given as a time varying feedback law affine in state:

$$u^*(t) = -R_i^{-1}B_i^T K_i^{\infty}(\tilde{x}(t) + \alpha_i) \quad \text{for} \quad r(t) = i \quad (18)$$

where the $\alpha_i(t)$ bias vector evolves according to:

$$\dot{\alpha_i}(t) = \left(A + K_i^{-1}Q_i\right)\alpha_i(t) + \sum_{j=1}^{3} \pi_{ij} K_i^{-1}K_j(\alpha_i(t) - \alpha_j(t) - q_{ij}) \quad (19)$$

$$\alpha_i(t_f) = 0 \quad i = 1,2,3$$

The bias $\alpha_i(t)$ relative to the standard solution comes from the fact that jumps coincide with state discontinuities obtained by Eq. (17). The α_i's can be obtained from Eq. (19) considered in steady state once Eq. (12) is solved.

In the next section, we provide the Markov model using the real data plus the obtained simulation results.

IV. Example System and Results

A. Example System Specifications

In this section, we will consider a system that conforms to the model described in the previous section and examine the simulation results. The load pertains to a small U.S. household, one similar in size and energy usage to common off-the-grid homes. The peak demand of 0.86 kW occurs in the evening and the mean usage is 0.49 kW. For this simulation, the initial hour is 6 pm. We assume no sun until 6 am.

The system has unique state-space matrices, $[A_i, B_i]$, for each mode $r(t)$. It will operate in a continuous state with $r(t) = i$ until the next mode, $r(t) = j$, occurs. During the night $P_i = 0$ and the system switches to a deterministic structure.

Among the components of the nGrid, we identified the solar contribution as the most influential. During a sunny day, the solar production completely satisfies the demand with some excess energy that can be stored in the battery for later use. The fuel cell was designed to satisfy the demand when the battery and solar could not. Lastly, for the household in this example, we employed a 7 kWh battery and imposed capacity limits at $\underline{E}_b = 1\,kWh$ and $\overline{E}_b = 6\,kWh$ to ensure maximum healthy lifetime of the device. We chose the initial state of charge of the battery to be 3.5 kWh.

B. Monte Carlo Simulation Results

In the following, we present results for a 5-day period. The cloud coverage was generated using the Markov chain transition matrix explained in Section II. Figure 3 presents household load data with simulated system response (top graph) and simulated cloud coverage (bottom graph).

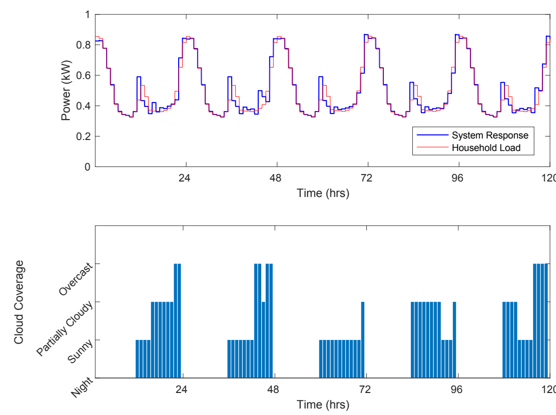

Fig. 3. System response with simulated response (top) and simulated cloud coverage (bottom).

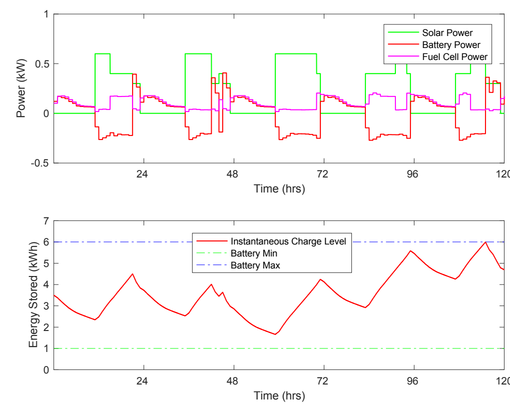

Fig. 4. Solar, battery, and fuel cell contributions (long overcast and night period).

Figure 4 shows the optimal control law (top graph) obtained from Eq. (18). During overcast weather and nighttime, the controller successfully compensates for the decreased solar generation by discharging the battery to match the load. The solar generation is a stochastic variable following the Markovian process described earlier. Depending on the weather, solar generation takes a constant value for that hour. The battery power takes a negative value during sunny and partially cloudy weather, which indicates that the battery is charging. Likewise, when the battery and fuel cell generation values are positive, they are discharging to satisfy the demand. The total amount of energy stored in the battery is shown in Figure 4 (bottom graph). The battery's maximum and minimum allowed levels are indicated by the horizontal lines. An item worth noting is that during the day, depending on solar generation which is piecewise constant in each mode, the battery and fuel cell may generate too much or not enough energy. This is due to two factors. First, for simplicity, we assumed that the set point is constant for one hour, since regulation is simpler than tracking for JLQ control. Second, the values of R_i and Q_i at each mode are not optimized since they are obtained by trial and error. In future work, we will use the theory of stochastic processes and their first passage time theory to develop a rapid tuning of the control law.

Due to the mostly sunny weather of this 5-day simulation, the battery never reaches the minimum allowed charge level (as seen in Figure 4). The battery's total charge trends upward over the 5-day period.

V. Conclusion And Future Work

This paper presents the application of jump linear quadratic control to the energy management problem of an islanded microgrid - nGrid. The nGrid utilizes solar and fuel cell generation combined with a battery energy storage system to satisfy the load of a small household. The solar generation is modeled as a continuous Markov process.

Simulation results for an example system showed the effectiveness of the proposed control method to track the load profile of the house. The control method was able to accommodate the stochastic jumps between operation modes and satisfy the demand each night. This control scheme paired with the presented microgrid architecture proved to be a robust and efficient implementation for grid islanded operations.

Future work will include more accurate battery and fuel cell models in order to better predict their generation output. Furthermore, in an effort to eventually compensate for the fact that the real problem is state constrained, we will use the theory of stochastic processes and their first passage-times to develop approximate theoretical expressions of battery mean times to first overcharge or first full discharge as a function of the performance measure parameters used in deriving the control law. This knowledge is subsequently used in achieving a rapid tuning of that law.

Acknowledgment

The authors would like to thank Professor Aleksandar Zecevic for his support and insightful discussions.

References

[1] Residential Solar Statistics – California Solar Initiative (CSI), www.gos. solarcalifornia.ca.gov

[2] X. Wu, X. Hu, S. Moura, X. Yin and V. Pickert, "Stochastic Control of Smart Home Energy Management with Plug-in Electric Vehicle Battery Energy Storage and Photovoltaic Array," *Journal of Power Sources*, vol. 333, pp. 203-212, Sept. 2016.

[3] A. Belloni, L. Piroddi, and M. Prandini, "A Stochastic Optimal Control Solution to the Energy Management of a Microgrid with Storage and Renewables," *2016 American Control Conference*, Boston, MA, 2016, pp. 2340-2345.

[4] Y. Ji, J. Wang, S. Yan, W. Gao and H. Li, "Optimal Microgrid Energy Management Integrating Intermittent Renewable Energy and Stochastic Load," *IEEE Advanced Information Technology, Electronic and Automation Control Conference,* Dec. 2015, pp. 334-338.

[5] J. Dong, A. Malikopoulos, S. Djouadi and T. Kuruganti "Application of Optimal Production Control Theory for Home Energy Management in a Microgrid," *2016 American Control Conference*, Boston, MA, 2016, pp. 5014-5019.

[6] S. Liu; X. Wang; P. X. Liu, "A Stochastic Stability Enhancement Method of Grid-Connected Distributed Energy Storage Systems," *IEEE Transactions on Smart Grid*, vol. PP, no. 99, pp. 1-9, 2017.

[7] B. Nordman and K. Christensen, "Local Power Distribution with Nanogrids," *Proceedings of the international Green Computing Conference*, June 2013.

[8] M. Mariton, *Jump Linear Systems in Automatic Control*, New York, Marcel Dekker Inc., 1990.

[9] Residential Load Modeling – System Advisor Model (SAM), National Renewable Energy Laboratory (NREL), sam.nrel.gov

[10] D. Sworder, R. Rogers, "An LQ-Solution to a Control Problem Associated with a Solar Thermal Central Receiver," *IEEE Trans. on Automatic Control*, vol. AC-28, no. 10, Oct. 1983.

[11] Meteorological Aviation Routine Weather Reports (METAR/SPECI), San Jose International Airport (SJC), www.noaa.gov

[12] J. Barton and D. Infield, "Energy Storage and Its Use with Intermittent Renewable Energy," *IEEE Trans. on Energy Conversion*, vol. 19, no. 2, pp. 441-448, June, 2004.

[13] Y. Ji and H.J. Chizeck, "Controllability, Stabilizability and Continuous-time Markovian Jump Linear Quadratic Control," *IEEE Trans. on Automatic Control*, vol. 35, no.7, July 1990.

[14] H. Abou-Kandil, G. Freiling, and G. Jank, "Solution and Asymptotic Behavior of Coupled Riccati Equations in Jump Linear Systems," *IEEE Trans. on Automatic Control*, vol. 39, no. 8, Aug. 1994.

[15] M. Khanbaghi, R. Malhamé and M. Perrier, "Optimal White Water and Broke Recirculation Policies in Paper Mills Via Jump Linear Quadratic Control," *IEEE Trans. on Control Systems Technology*, vol. 10, no. 4, July 2002.