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Online Pricing Mechanisms for Electric Vehicle Management at Workplace Charging Facilities

Nathaniel Tucker Mahnoosh Alizadeh

Abstract—**In this paper, we design posted price mechanisms for assigning electric vehicles (EVs) to electric vehicle supply equipment (EVSEs) to maximize the smart charging potential of a workplace parking facility. Our approach can accommodate diverse smart charging objectives for the facility, e.g. minimize electricity costs from time-of-use rates, maximize behind-the-meter solar integration, or provide ancillary services. We also accommodate individual users' preferences for specific charging spots. To optimize the assignment of EVs to EVSEs, we develop three different pricing heuristics that allow each arriving EV to solve for its own optimal (i.e., cost minimizing) EVSE assignment. The first approach is based on the use of online convex optimization (OCO) to learn the EVSE prices and performs price updates after each arrival. We discuss performance guarantees for the OCO approach compared to the optimal offline solution. The second and third approaches make use of Lagrangian relaxation to decompose a modified multi-user problem into many single-user problems of small dimension. These two approaches provide EVSE prices that are only updated once per day. The second approach uses a stochastic supergradient based method to converge to optimal daily EVSE prices and can only accommodate stationary charge statistics. The third approach makes use of a one-shot learning technique to learn the prices each day, which allows for non-stationary charge request statistics.**

I. Introduction

In recent years, sales of both battery electric vehicles (BEVs) and plug-in hybrid electric vehicles (PHEVs) have increased rapidly. In the U.S., the annual sales of these EVs have grown by over 700% since 2011 [1]. Furthermore, technology improvements in both the EVs and EVSEs have decreased the average charging time to 4.8 hours for BEVs and 2.8 hours for PHEVs [2]. With the increasing number of vehicles with short charging times, *smart charging* techniques such as load shifting and demand response have potential to grow significantly.

Specifically, there is much potential at workplaces with large parking infrastructures where consistently high numbers of EVs park and charge for the duration of the workday. Currently, many workplaces with charging infrastructure make use of Single-Output-Single-Cable (SOSC) EVSEs where one EV plugs into one EVSE for an unknown duration of the workday. In this case, the EVSE immediately begins charging the EV after being plugged in and continues until the EV's battery is fully charged or the EV departs. Additionally, the employee that owns the EV may or may not remove his EV from the EVSE during the workday to allow other employees usage. Obviously, this system is inefficient for the owners of the EVs if there is high demand of the EVSEs. Moreover, the facility management is unable to benefit from smart charging opportunities due to the inability to control charging schedules.

To improve the system for both the EV owners and facility management, we assume the parking structure can be equipped with one of the two following solutions: 1) Single-Output Multiple-Cable (SOMC) EVSEs may be used in lieu of SOSC EVSEs [3]. Each SOMC EVSE can be connected to multiple EVs but only charges one EV at a time (Single Output). This enables facility management to devise a smart charging plan for each EVSE for the duration of the day, while satisfying the charging needs of all EVs; 2) Alternatively, a reservation based mechanism can be implemented on SOSC EVSEs. In this case, EVs make reservations at the beginning of the day to utilize a certain SOSC EVSE during a specific timeslot. Having full information about the energy demand each EVSE needs to satisfy, this second solution will also enable facility management implement smart charging solutions such as load shifting and providing ancillary services to the local grid.

A number of past studies have proposed smart charging implementations for large populations of EVs. In both [4] and [5], large sets of EVs are used for the purpose of frequency regulation. Similarly, [6] presents a parking lot system with EVSEs and solar generation capabilities for capacity enhancement of the distribution system. The authors in [7] present an aggregation method for large numbers of distributed EVs for ancillary service provision. Reviews of the many services and benefits that can be provided by smart charging and EV fleet aggregation can be found in [8] and [9]. Besides the smart charging implementations, there has been research on the assignment of EVs to EVSEs. The authors in [10] utilized a game theoretic pricing method to guide EVs to EVSEs. However, their goals were aimed at minimizing congestion and optimizing utilization of power evenly across the network of EVSEs. Additionally, [11] presented centralized assignment heuristics for EVs to EVSEs within a workplace parking structure. The paper closest to ours is [3], where the concept of an SOMC EVSE was presented; however, a greedy heuristic was used in their implementation to assign EVs to EVSEs.

N. Tucker and M. Alizadeh are with the Department of Electrical and Computer Engineering, University of California, Santa Barbara. nathaniel_tucker@ucsb.edu.

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In this paper, we design posted price mechanisms for assigning arriving EVs to EVSEs with the goal of maximizing the parking structure's smart charging potential. Our approach can accommodate many diverse smart charging objectives for the facility, e.g. minimize electricity costs from time-of-use rates, maximize behind-the-meter solar integration, or provide ancillary services. To give a simple example of the benefits of smart assignment, consider a workplace where all of the arriving employees with EVs prefer to use the EVSEs that are closest to the building entrance. Without an assignment mechanism, the EVSEs near the building entrance will have large total energy requests and cannot provide much smart charging benefit. The same holds for the EVSEs farther from the entrance with few EVs plugged in. To create a better assignment of EVs to EVSEs, we develop three different pricing heuristics that allow each arriving EV to solve for its own optimal (i.e., cost minimizing) EVSE assignment. The first approach is based on the use of online convex optimization to learn the EVSE prices and performs price updates after each arrival. The second and third approaches make use of a Lagrangian relaxation to decompose a modified multi-user problem into many single-user problems of small dimension. The latter two approaches provide EVSE prices that are only updated once per day.

The remainder of this paper is organized as follows. Section II presents the system structure and introduces the problem formulation. Section III presents our OCO heuristic with its performance guarantee. Section IV describes the modified problem formulation for dual decomposition with the Lagrangian relaxation to provide EVSE prices that are updated once per day. Section V presents an extension to the approach from Section IV by adding a one-shot learning technique for the EVSE prices each day. Section VI discusses the effects of rounding. Section VII presents numerical results to validate the proposed heuristics.

II. Problem Formulation

A. EVs and EVSEs—Problem notation

In this section, we describe the attributes of the arriving EVs and the EVSEs in the parking structure. Each EVSE in the structure has index $b \in \mathcal{B} = \{1, \dots, B\}$. EV arrival events are indexed by $r \in \mathcal{R} = \{1, \dots, R\}$. The amount of energy requested by arrival r on each day $k = 1, \dots, K$ is denoted as $E^{(r,k)}$. Each EV arrives with a feasible subset of EVSEs that it would like to use, denoted as $F^{(r,k)} \subseteq \mathcal{B}$. We assume that for each arrival r, $E^{(r,k)}$ and $F^{(r,k)}$ are sampled i.i.d. values. For each arrival r, the EV is assigned to EVSEs; this assignment takes the form of a $0, 1$ assignment vector $\vec{x}^{(r,k)}$ of dimension $B \times 1$. The total amount of energy requested by the EVs connected to EVSE b is denoted by $\ell_b^{(k)} = \sum_{r=1}^{R} E^{(r,k)} x_b^{(r,k)}$ and is referred to as the level of the EVSE.

B. State dynamics and problem formulation

In this section, we present the state dynamics and problem formulation for optimally assigning EVs to EVSEs as they arrive in the morning. The state of the structure at stage $r = 1, \dots, R$ (after the arrival of EV r) on day k is fully described by the levels of the EVSEs $\vec{\ell}^{\,(r,k)}$. At the start of each day k (i.e. $r = 1$), $\vec{\ell}^{\,(1,k)}$ is initialized at zero and increases as each EV r parks within the structure. It is important to note that we assume all employees arrive in the morning and park their EV for the duration of the workday (i.e., there are no early departures or late arrivals). Additionally, we assume all R EVs arrive in a short time period every morning. As such, the inter-arrival times are insignificant compared to the length of the workday. Due to the negligible inter-arrival times and the assumption that there are no early departures, the total operational cost incurred in order to serve the charge requests of all EVs in the parking infrastructure is a function of the levels of the EVSEs after the last EV arrives for the day. Considering smart charging opportunities, we assume that the daily operational cost is a strongly convex function $C_{\mathcal{B}}(\vec{\ell}^{\,(k)})$ of the EVSE levels $\vec{\ell}^{(k)} = [\ell_b^{(k)}]_{b \in \mathcal{B}}$. With strong convexity being the only restriction on $C_{\mathcal{B}}(\vec{\ell}^{\,(k)})$, this objective function can be designed for many objectives like energy costs or prioritizing behind-the-meter renewable integration.

Let us assume that the facility manager has access to statistics of the arrivals' energy requests and feasible EVSE subsets. Theoretically, the problem of assigning EVs to EVSEs as they arrive can be formulated as a dynamic program where the entire cost is realized at the final stage, when the R-th arrival enters the parking structure for the day. This can also be formulated as a stochastic optimization problem $P^{(k),I}$ for each day k:

$$\min_{\substack{\vec{x}^{(r,k)} \in F_\xi^{(r,k)},\ \ell_b^{(k)} | I^{(r-1,k)} \\ r=1,\dots,R}} \ \mathbb{E}_{\xi} \left[C_{\mathcal{B}}(\vec{\ell}_\xi^{\,(k)}) \right] \tag{1a}$$

$$\text{s.t.} \quad \ell_{b,\xi}^{(k)} = \sum_{r=1}^{R} E_\xi^{(r,k)} x_b^{(r,k)} \quad \forall b \in \mathcal{B} \tag{1b}$$

$$\sum_{b=1}^{B} x_b^{(r,k)} = 1 \quad \forall r \in \mathcal{R} \tag{1c}$$

$$x_b^{(r,k)} \in \{0, 1\} \quad \forall b \in \mathcal{B},\ r \in \mathcal{R}. \tag{1d}$$

The objective function captures the expected cost of the parking facility over the set of different scenarios $\xi \in \Xi$ that may happen given the random nature of the EV charge requests $E^{(r,k)}$ and their set of feasible EVSEs $F^{(r,k)}$. Constraint (1b) defines the energy request level for each EVSE. Constraint (1c) states that each EV has to be allocated to an EVSE each day. Constraint (1d) is an integer constraint on the assignments as EVs cannot be split among different EVSEs. Moreover, due to the online nature of the problem, each assignment is made with a nonanticipativity constraint. We denote $I^{(r,k)}$ as the information collected which is available when selecting an assignment at stage $r + 1$ on day k. This information represents the collection of all previous assignments and energy levels of the parking structure. This information evolves as EVs arrive each day, as $I^{(r,k)} = I^{(r-1,k)} \cup \{\vec{x}^{(r,k)}, \vec{\ell}^{(r,k)}\}$.

If the optimization problem in (1a)-(1d) can be directly solved then one can derive a shadow pricing mechanism that updates posted prices after each EV arrival. However, exact solutions suffer from dimensionality issues as the number of EVs and EVSEs increases. Furthermore, the statistics of energy requests and feasible EVSEs might be unavailable, unreliable, or vary from day to day. As such, deriving a posted price mechanism by directly solving the stochastic optimization problem is both intractable as the number of states grows as well as potentially inaccurate due to its dependency on arrival statistics.

Accordingly, we propose posted price heuristics to allow the EVs to solve for their own allocations without suffering from these aforementioned issues. Our approach relies on convex programming techniques. Hence, for the purposes of this paper, we relax the integer constraint in the following sections and allow fractional allocations (with cost $P^{(k),F}$). We will discuss the effects of this relaxation on the quality of our solutions and rounding techniques in Section VI.

III. Online Stochastic Convex Optimization for Dynamically Updated Prices

In this section, we present an online primal-dual algorithm for posted prices and discuss performance guarantees compared to the optimal offline solution (i.e., a solution that has access to all EV arrival information at the beginning of the day). Our algorithm makes use of the learning paradigm of online convex optimization. In the OCO framework, one considers a sequence of stages where at each stage a learner selects a vector from a convex set that affects their cost. In many cases, the learner's choice is compared to the optimal choice in hindsight, and the learner suffers a loss dependant on this difference. Our algorithm makes use of primal-dual OCO techniques to predict the dual variables at each stage to be used as EVSE prices. Using OCO to predict dual variables provides an efficient method to update EVSE prices after each arrival and allows our algorithm to accept input sampled from unknown distributions that vary from day to day.

Our posted price heuristic for online EV assignment with price updates occurring at each state transition is based off the OCO framework presented in [12]. The framework in [12] is as follows: Consider an online problem where the objective is to maximize a concave function that is dependent on the average assignment (instead of each individual assignment) such as $\max f(\vec{x}^{\,avg}) = \max f(\frac{1}{R}\sum_{r=1}^{R} \vec{x}^{(r)})$. The main issue is the separability of the problem. If the problem was $\max \frac{1}{R}\sum_{r=1}^{R} f^{(r)}(\vec{x}^{(r)})$, the solution is simple, at each stage set $x^{(r)} = \arg\max_{x^{(r)}} f^{(r)}(\vec{x}^{(r)})$. The issue arises from not being able to know the contribution of $\vec{x}^{(r)}$ to the entire objective due to the non-anticipativity constraint. By use of Fenchel duality, the objective function can be linearized as $f(\vec{x}^{\,avg}) = f^*(\vec{\lambda}^*) - \frac{1}{R}\sum_{r=1}^{R} \vec{\lambda}^* \vec{x}^{(r)}$ for some $\vec{\lambda}^*$ in hindsight, where $f^*()$ denotes the Fenchel conjugate of $f()$. This is defined as $f^*(\vec{\lambda}) := \max_{\vec{y}}\{\vec{y}^{\,T}\vec{\lambda} - f(\vec{y})\}$. Provided with a prediction $\vec{\lambda}^{(r)}$ of the dual variable $\vec{\lambda}^*$, the linearized objective function can be solved at each stage and the dual variable is then updated for the next stage using an OCO technique.

In order to utilize this framework, we slightly restructure our stochastic optimization problem $P^{(k),F}$ through a change of variable. First, we are no longer using the assignment vector $\vec{x}^{(r,k)}$; instead, we decide the amount of energy that should be provided to each EV by each EVSE. This is done through a simple mapping $\vec{v}^{(r,k)} = \vec{x}^{(r,k)} E^{(r,k)}$. Second, we define the average charge assignment as $\vec{v}^{\,(avg,k)} = \frac{1}{R}\sum_{r=1}^{R} \vec{v}^{(r,k)}$. Third, we use a modified cost function $C'_{\mathcal{B}}(\vec{v}^{\,(avg,k)}) = C_{\mathcal{B}}(R\vec{v}^{\,(avg,k)})$. With this notation, the problem formulation (1a)-(1d) on day k is reformulated to $P^{(k),F}_{OCO}$:

$$\max_{\substack{\vec{v}^{\,(r,k)},\, \vec{v}^{\,(avg,k)}|I^{(r-1,k)} \\ r=1,\dots,R}} \Big[- C'_{\mathcal{B}}(\vec{v}^{\,(avg,k)}) \Big] \tag{2a}$$

$$\text{s.t.} \quad \vec{v}^{\,(avg,k)} = \frac{1}{R}\sum_{r=1}^{R} \vec{v}^{\,(r,k)} \tag{2b}$$

$$\sum_{b=1}^{B} v_b^{(r,k)} = E^{(r,k)} \quad \forall r \in \mathcal{R} \tag{2c}$$

$$\frac{1}{E^{(r,k)}} \vec{v}^{\,(r,k)} \in F^{(r,k)} \quad \forall r \in \mathcal{R}. \tag{2d}$$

Our posted price algorithm, which updates the EVSE prices after each EV's arrival, is presented in Algorithm 1.

Algorithm 1 OnlineConvexAllocation(R)

Initialize $\vec{\lambda}^{(1)}$
for $r = 1 : R$ **do**
  EV r chooses assignment:
  $\vec{v}^{(r),\dagger} = \arg\max_{\vec{v}^{(r)}} \big\{ - \vec{\lambda}^{(r)} \vec{v}^{(r)} \big\}$
  Central controller chooses $\vec{\lambda}^{(r+1)}$ by doing an OCO update for:
  $\psi^{(r)}(\vec{\lambda}) = \vec{\lambda}\vec{v}^{\,(r),\dagger} - \max_{\vec{y}} \big\{ \vec{y}\vec{\lambda} - C'_{\mathcal{B}}(\vec{y}) \big\}$
end for

This algorithm provides each incoming EV with a tractable optimization problem to solve for their own assignment as well as an efficient method for price updates at state transitions. Intuitively, when an EV chooses EVSE b, the price, λ_b should increase to dissuade congestion at that EVSE. Similarly, if an EVSE is underutilized, the corresponding price should be decreased to promote usage. Each time a new arrival is allocated, the OCO learning algorithm observes loss defined as $\psi^{(r)}(\vec{\lambda}) = \vec{\lambda}\vec{v}^{\,(r),\dagger} - \max_{\vec{y}} \big\{ \vec{y}\vec{\lambda} - C'_{\mathcal{B}}(\vec{y}) \big\}$. This loss represents the regret from the current posted prices against the optimal posted prices in hindsight. With the observed loss, an OCO algorithm such as online gradient descent or the multiplicative weight method can be applied to calculate the next price vector [13]. Specifically, the multiplicative weights update method is fast and efficient. Given that the OCO loss function is bounded, $0 \leq \psi^{(r)}(\vec{\lambda}) \leq M$, and a parameter $\epsilon > 0$, the new EVSE prices can be calculated as:

$$\lambda_b^{(r+1)} = \frac{w_b^{(r)}}{\sum_b w_b^{(r)}}, \text{ where } w_b^{(r)} = w_b^{(r-1)}(1+\epsilon)^{\psi^{(r)}(\vec{e}^{\,b})/M}. \tag{3}$$

When considering the performance of Algorithm 1, we compare it to the optimal offline solution. We denote the optimal offline cost on day k as $OPT^{(k)}$. We use the concept of regret as our performance metric. That is, the amount of additive error of our algorithm's cost on day k compared to $OPT^{(k)}$. With the reformulation of the online stochastic assignment problem in (2a)-(2d), the regret result from [12] can be used as follows: After all R EVs have arrived on day k, the average difference between the optimal offline solution and our heuristic is:

$$\begin{aligned} \mathbb{E}[avg\text{-}regret(R)] &:= \mathbb{E}[(OPT^{(k)} - C'_{\mathcal{B}}(\vec{v}^{\,(avg,k)}))] \\ &= O\bigg(\sqrt{\frac{B \log B}{R}}\bigg). \end{aligned} \tag{4}$$

IV. Lagrangian Relaxation for Daily Resource Prices

In some applications, it could be advantageous to have prices that do not update after each arrival, contrasting the OCO solution presented in Section III. Our first proposed solution for learning the optimal *state-independent* prices is appropriate for a workplace where the arriving EVs have EVSE preferences and energy requests that are sampled i.i.d. values from distributions that do not vary from day to day. We will discuss an alternative solution that can function given distributions that vary on a daily basis in Section V.

In this section, we use Lagrangian relaxation and dual decomposition methods to design a posted price heuristic for daily EVSE prices. However, in order to guarantee convergence for a distributed solution, we modify our objective function in (1a) by adding strongly convex user cost functions. We then utilize a stochastic supergradient technique to update the EVSE prices after each day. As more days pass, the algorithm will continue to post improved prices until the optimal prices are discovered.

A. Lagrangian relaxation

In order to utilize dual decomposition, we modify (1a) by removing the restricted EVSE subset $F^{(r,k)}$ for each arrival. Instead, arrivals experience a discomfort cost proportional to the distance between their assigned EVSE and their desired EVSE (denoted as $\vec{x}^{(r,pref)}$). We denote the strongly convex cost of discomfort to EV r on day k due to the distance of allocation $\vec{x}^{(r,k)}$ to the desired spot $\vec{x}^{(r,pref)}$ as $C_r(\vec{x}^{(r,k)})$. Additionally, the sum $\sum_{r=1}^{R} C_r(\vec{x}^{(r,k)})$ represents the discomfort cost incurred by all arrivals due to their EVSE assignments on day k.

The modified primal can be written as:

$$\min_{\substack{\vec{\ell}_\xi^{(k)},\, \vec{x}_\xi^{(r,k)} | I^{(r-1,k)} \\ r=1,\dots,R}} \mathop{\mathbb{E}}_{\xi} \Big[C_{\mathcal{B}}(\vec{\ell}_\xi^{(k)}) + \sum_{r=1}^{R} C_r(\vec{x}_\xi^{(r,k)}) \Big] \tag{5a}$$

$$\text{s.t.} \quad \ell_{b,\xi}^{(k)} = \sum_{r=1}^{R} E_\xi^{(r,k)} x_{b,\xi}^{(r,k)} \quad \forall b \in \mathcal{B} \tag{5b}$$

$$\sum_{b=1}^{B} x_{b,\xi}^{(r,k)} = 1 \quad \forall r \in \mathcal{R}. \tag{5c}$$

Solving (5a)-(5c) and setting EVSE prices as the Lagrange multipliers of the respective dual suffers from the same issues mentioned in Section II-B. An exact solution to (5a)-(5c) is both intractable as the number of states grows and requires knowledge of the arrival statistics. The dual problem with daily Lagrangian multipliers on day k is given in (6):

$$\begin{aligned} \max_{\vec{\lambda}_\xi^{(k)}} \min_{\substack{\vec{\ell}_\xi^{(k)},\, \vec{x}_\xi^{(r,k)} \\ | I^{(r-1,k)} \\ r=1,\dots,R}} \mathop{\mathbb{E}}_{\xi} \Big\{ & C_{\mathcal{B}}(\vec{\ell}_\xi^{(k)}) + \sum_{r=1}^{R} C_r(\vec{x}_\xi^{(r,k)}) \\ & + \sum_{b=1}^{B} \lambda_{b,\xi} \Big(\Big(\sum_{r=1}^{R} E_\xi^{(r,k)} x_{b,\xi}^{(r,k)} \Big) - \ell_{b,\xi}^{(k)} \Big) \Big\}. \end{aligned} \tag{6}$$

Solving for the optimal scenario-dependent Lagrange multipliers in (6) amounts to solving the original DP in (5a)-(5c). In order to reduce computational complexity, here we impose a relaxation on the Lagrangian multipliers across all scenarios. Specifically, we force uniform Lagrangian multipliers across all scenarios: $\vec{\lambda}_\xi = \vec{\lambda}^\star \; \forall \xi \in \Xi$. This allows the central controller to calculate one set of prices each morning (independent of scenario) and leave them for the whole workday. This Lagrangian relaxation approach has been studied in papers such as [14]–[17]. The resulting dual problem is:

$$\begin{aligned} \max_{\vec{\lambda}} \min_{\substack{\vec{\ell}_\xi^{(k)},\, \vec{x}_\xi^{(r,k)} \\ | I^{(r-1,k)} \\ r=1,\dots,R}} \mathop{\mathbb{E}}_{\xi} \Big\{ & C_{\mathcal{B}}(\vec{\ell}_\xi^{(k)}) + \sum_{r=1}^{R} C_r(\vec{x}_\xi^{(r,k)}) \\ & + \sum_{b=1}^{B} \lambda_b \Big(\Big(\sum_{r=1}^{R} E_\xi^{(r,k)} x_{b,\xi}^{(r,k)} \Big) - \ell_{b,\xi}^{(k)} \Big) \Big\}. \end{aligned} \tag{7}$$

B. Relaxed primal problem

Before proposing our pricing mechanism, let us discuss the effects of the relaxation (7) on our problem. By imposing uniform Lagrange multipliers across all possible daily scenarios, the dual problem presented in (7) is no longer the dual to the primal problem (5a)-(5c). Rather, it is the dual to the new primal problem (8a)-(8c):

$$\min_{\substack{\vec{\ell}_{\xi}^{(k)},\, \vec{x}_{\xi}^{(r,k)}|I^{(r-1,k)} \\ r=1,\dots,R}} \mathbb{E}_{\xi}\left[C_{\mathcal{B}}(\vec{\ell}_{\xi}^{(k)}) + \sum_{r=1}^{R} C_r(\vec{x}_{\xi}^{(r,k)})\right] \quad (8a)$$

$$\text{s.t.} \quad \mathbb{E}_{\xi}\left[\ell_{b,\xi}^{(k)}\right] = \mathbb{E}_{\xi}\left[\sum_{r=1}^{R} E_{\xi}^{(r,k)} x_{b,\xi}^{(r,k)}\right] \quad \forall b \in \mathcal{B} \quad (8b)$$

$$\sum_{b=1}^{B} x_{r,\xi}^{b,(k)} = 1 \quad \forall r \in \mathcal{R}. \quad (8c)$$

Comparing (8a)-(8c) to (5a)-(5c), we note one difference: the constraint (8b) holds only in expectation. That is, constraint (5b) will not be upheld for some scenarios, but will hold when averaging across all scenarios.

Given the optimal dual variables $\vec{\lambda}^{\star}$, the relaxed problem can be decomposed into $R+1$ smaller problems and solved independently. On arrival each day k, EV r is required to solve

$$\vec{x}^{(r,k),\dagger} = \arg\min_{\vec{x}^{(r,k)}} \left\{C_r(\vec{x}^{(r,k)}) + E^{(r,k)}\left([\vec{\lambda}^{\star}]^T \vec{x}^{(r,k)}\right)\right\} \quad (9)$$

in order to find its EVSE assignment for the day. Moreover, the central controller solves (10):

$$\vec{\ell}^{(k),\ddagger} = \arg\min_{\vec{\ell}^{(k)}} \left\{C_{\mathcal{B}}(\vec{\ell}^{(k)}) - [\vec{\lambda}^{\star}]^T \vec{\ell}^{(k)}\right\}. \quad (10)$$

C. Stochastic supergradient price updates

We now present an algorithm to find $\vec{\lambda}^{\star}$ in the absence of statistics on the EVs arrivals. Denote the price posted on day k of the algorithm as $\vec{\lambda}^{(k)}$. We will propose a stochastic supergradient based method to update the daily prices. We assume that after each EV r arrives and solves (9), it submits $\vec{x}^{(r,k),\dagger}$ to the central EVSE controller. These values can be used to calculate the realized energy levels of the EVSEs denoted as $\vec{\ell}^{(k),\dagger} = E^{(r,k)}\vec{x}^{(r,k),\dagger}$. Note that this is not equivalent to the energy levels of EVSEs on day k determined through (10). This decentralized approach can be seen in Algorithm 2.

The supergradient of (7) with respect to $\vec{\lambda}^{\star}$ is given by

$$\vec{z}_{\lambda}^{(k)} = \mathbb{E}_{\xi}\left(\vec{\ell}_{\xi}^{(k),\dagger} - \vec{\ell}_{\xi}^{(k),\ddagger}\right). \quad (11)$$

Instead of using a standard supergradient method and evaluating the expectation in (11) across all scenarios, we use the realized sample path as an unbiased estimate of the supergradient. That is, for the simulated sample path for $k = 1, \dots, K'$ the constraint violation $\tilde{z}_{\lambda}^{(K')} = \frac{1}{K'}\sum_{k=1}^{K'}[\vec{\ell}_{\xi}^{(k),\dagger} - \vec{\ell}_{\xi}^{(k),\ddagger}]$ is an unbiased estimate of a supergradient of the relaxed dual at K'. To reduce the variance of the supergradient estimate, batch gradient averaging across many sample paths could be implemented [18]. The Lagrange multipliers are updated at the end of each day k as follows:

$$\vec{\lambda}^{(k+1)} = \vec{\lambda}^{(k)} + \alpha^{(k)}\tilde{z}_{\lambda}^{(k)}. \quad (12)$$

where $\tilde{z}_{\lambda}^{(k)}$ is an unbiased estimate of the supergradient. Additionally, $\alpha^{(k)}$ is a diminishing step-size satisfying $\sum_{i=1}^{\infty}\alpha^{i} = \infty$ and $\sum_{i=1}^{\infty}(\alpha^{i})^2 < \infty$. A common implementation is $\alpha^{(k)} = \frac{1}{k}$. We can achieve faster convergence to the optimal prices by initializing after observing a few days of EV arrivals. That is, record the mean capacity requests and EVSE preferences and initialize the prices using this data.

Algorithm 2 LAGRANGIANRELAXATION(R)

```
1: Initialize λ⃗^(1) for day k = 1
2: for k = 1 : K do
3:     for r = 1 : R do
4:         EV r chooses assignment:
           x⃗^(r,k),† = arg min_{x⃗^(r,k)} { C_r(x⃗^(r,k)) + E^(r,k)([λ⃗^(k)]^T x⃗^(r,k)) }
5:         EV r submits x⃗^(r,k),† to the central controller
6:     end for
7:     Central controller solves:
       ℓ⃗^(k),‡ = arg min_{ℓ⃗^(k)} { C_B(ℓ⃗^(k)) − [λ⃗^(k)]^T ℓ⃗^(k) }
8:     Central controller updates the next day's prices:
       λ⃗^(k+1) = λ⃗^(k) + α^(k) z̃_λ^(k)
9: end for
```

V. ONE-SHOT LEARNING FOR DAILY RESOURCE PRICES

As seen in Section IV, the Lagrangian relaxation approach requires a stationary arrival distribution across days in order to converge to the optimal prices. A stationary arrival distribution may not be applicable to all parking structures. To post state-independent prices given non-stationary arrival distributions, we make use of a learning-based algorithm that works by "observing" EV arrivals for a short period of time each day and then setting state-independent EVSE prices for the rest of the day. This requires the first fraction of EVs to reveal all their parameters to the central control. Similar learning approaches have been studied extensively for online linear program applications with random input in [19], [20], and [21]. Moreover, [22] and [23] present similar results for online problems with concave functions. Specifically, [23] presents a one-shot learning algorithm that solves a carefully chosen partial allocation problem and uses the optimal solution to guide the future decisions. Utilizing a similar approach, our heuristic solves a partial problem using the first small fraction of arrivals to learn the near-optimal dual price vector. Once this is accomplished, the remaining large fraction of EVs allocate themselves using the learned dual prices. This approach only requires solving one optimization problem per day for a small fraction of the arrivals.

Let $s = \lceil \epsilon R \rceil$ be the fraction of arriving EVs that will be used to learn the dual price vector, $\vec{\lambda}^{(k)}$, where ϵ is a small fractional value ($0 < \epsilon < 1/2$). For the first $s = \lceil \epsilon R \rceil$ EVs, the daily prices have yet to be established; therefore, these first arrivals can either choose whatever EVSE they desire most or, if there is some correlation across days, the central controller can post the previous day's prices. Either option will result in a small loss. The goal is to select the value ϵ carefully: if it is too large, then too many EVs will be used in the learning phase, and the resulting operational cost will be high. If ϵ is too small, the sample set of EVs for the learning phase might not give a good estimate of the incoming distribution. Once the learning phase is complete, all subsequent arrivals $r > s$, will allocate themselves just as in Section IV. At stage s of each day k, the central controller uses the available information from the learning phase to solve the following partial problem:

$$\min_{\substack{\vec{\ell}_\xi^{(k)},\, \vec{x}_\xi^{(r,k)} \\ |I^{(s,k)} \\ r=1,\ldots,s}} \left\{ C_{\mathcal{B}}(\vec{\ell}_\xi^{(k)}) + \sum_{r=1}^{s} \frac{1}{\epsilon} C_r(\vec{x}_\xi^{(r,k)}) \right\} \tag{13a}$$

$$\text{s.t.} \quad \ell_{b,\xi}^{(k)} = \sum_{r=1}^{s} \frac{1}{\epsilon} E_\xi^{(r,k)} x_{b,\xi}^{(r,k)} \quad \forall b \in \mathcal{B} \tag{13b}$$

$$\sum_{b=1}^{B} x_{b,\xi}^{(r,k)} = 1 \quad \forall r \leq s. \tag{13c}$$

After solving this optimization, the central controller posts $\vec{\lambda}^{(k)}$ (the optimal dual variable for constraint (13b)) and the remaining EVs allocate themselves accordingly. With this one-shot learning technique, we allow for different distributions of arriving EVs each day. The one-shot approach can be seen in Algorithm 3.

Algorithm 3 ONE-SHOT-LEARNING ALLOCATION(ϵ, R)

1: $s = \lceil \epsilon R \rceil$
2: **for** $r = 1 : s$ **do**
3: Central controller observes arrival r's energy request and EVSE preferences
4: Allocate EV r arbitrarily
5: **end for**
6: Central controller sets $\vec{\lambda}^{(k)}$ as the optimal dual variable from (13a)-(13c)
7: **for** $r = s + 1 : R$ **do**
8: EV r chooses assignment: $\vec{x}^{(r,k),\dagger} = \arg\min_{\vec{x}^{(r,k)}} \left\{ C_r(\vec{x}^{(r,k)}) + E^{(r,k)} \left([\vec{\lambda}^{(k)}]^T \vec{x}^{(r,k)} \right) \right\}$
9: **end for**

VI. ONLINE ROUNDING

Since our proposed heuristics rely on convex programming techniques, we relaxed the integer constraint in the previous sections and allowed fractional allocations. However, in a physical parking structure, one cannot fractionally assign EVs to EVSEs. As such, rounding techniques must be used to generate feasible integer solutions. It is important to note that any rounding technique must also be online as the EVs need to be assigned as they arrive.

After arriving to the parking structure on day k, each EV r solves for a fractional assignment $\vec{x}^{(r,k),\dagger}$. An integer solution $\vec{x}^{(r,k),I}$ can be obtained using randomized rounding [24]. Specifically, round the fractional assignments $x_b^{(r,k),\dagger}$ to 1 with probability $x_b^{(r,k),\dagger}$ (and round $x_b^{(r,k),\dagger}$ to 0 with probability $1 - x_b^{(r,k),\dagger}$). The expected total operational cost of the randomized rounding solution is equal to the total operational fractional cost [25]. However, this approach can yield infeasible allocations (e.g., assigning the same EV to multiple EVSEs or not at all). To combat this issue, the rounding step can be repeated multiple times to ensure a feasible solution or other application specific deterministic rounding techniques can be used [26].

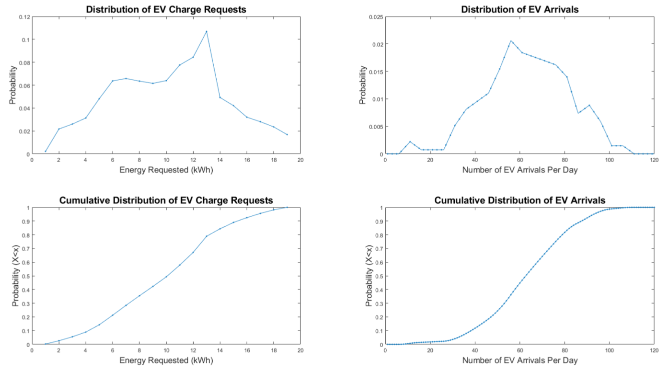

Fig. 1. Left Column: EV energy request distribution. Right Column: EV arrival distribution.

VII. SIMULATION RESULTS

In this section, we present simulation results highlighting the performance of each posted price heuristic. The EV energy request data used in the following simulations comes from charging sessions of a company in Mountain View, California. Only sessions started between 6:00am and 9:00am are included to represent arrivals at a workplace. For the following simulations, we generated daily scenarios based on the distributions in Figure 1.

A. Nonstationary arrivals

In this section, we compare the performance of our OCO heuristic and the optimal offline solution. We do not directly compare OCO with the Lagrangian relaxation or one-shot due to the addition of the user cost term in the objective function for the latter two approaches.

Figure 2 shows the daily operational cost for a parking structure where arrival EVSE preferences vary from day to day across 10 days. The OCO algorithm always performs close to the optimal solution. In Figure 2, the maximum regret between OCO and OPT occurs on day 2 where we get an average regret per arrival of 0.7603.

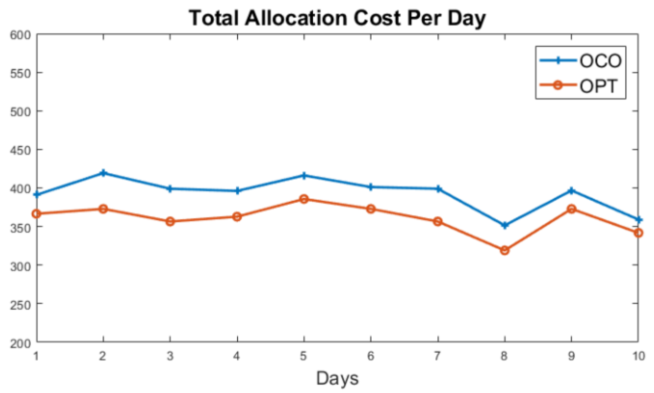

Fig. 2. Total parking structure allocation costs for nonstationary arrival preferences. $K = 10$, $R = 61$, $B = 16$.

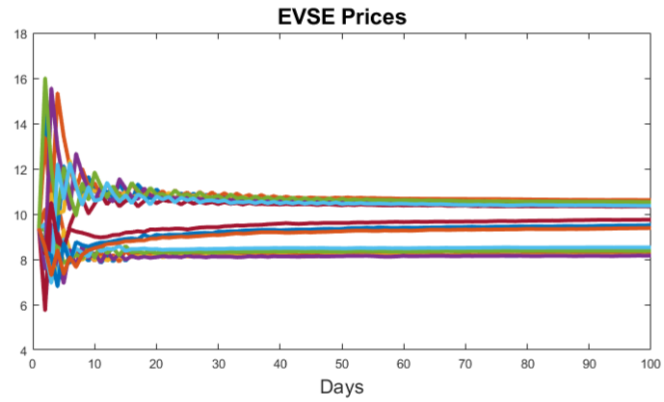

Fig. 4. Lagrangian relaxation EVSE price convergence. $K = 100$, $R = 61$, $B = 16$.

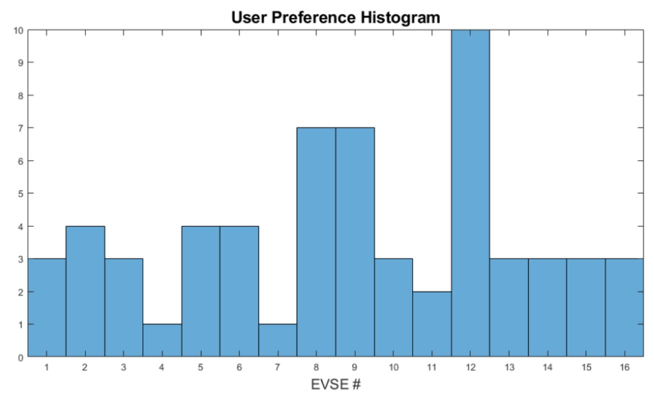

Fig. 3. Stationary user preference histogram for $R = 61$.

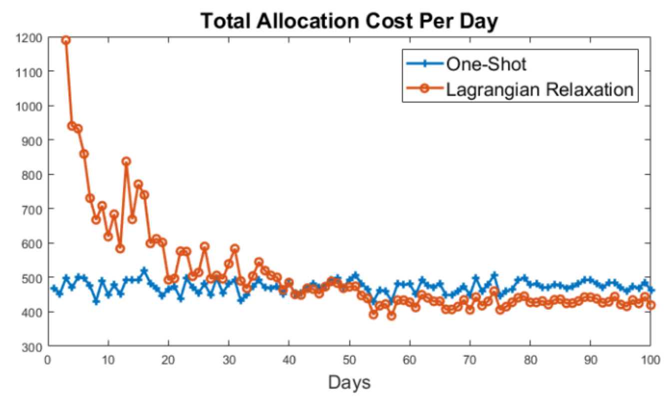

Fig. 5. Total parking structure allocation costs for stationary arrival preferences. $K = 100$, $R = 61$, $B = 16$.

B. Stationary arrivals

In this section, we compare the performance of one-shot and the Lagrangian relaxation heuristic for an average workplace where the same EVs arrive each day, with $C_r(.)$'s given for each employee r, e.g., close to their office location. The further the assignment, the higher the user's dissatisfaction. The histogram in Figure 3 presents the EVSE preferences that were used for this simulation. Only the highest preference is shown.

The simulation with stationary user preference inputs was performed over $K = 100$ days to allow the Lagrange relaxation solution's prices to converge. Each day, $R = 61$ arrivals enter the parking structure with $E^{(r,k)}$ sampled from the distribution presented in Figure 1.

Figure 4 shows the prices for all of the 16 EVSE's available on site converging for the Lagrangian relaxation algorithm. The first 20 days have large oscillations in pricing due to the algorithm exploring different prices. Once the prices have converged, each EV will use the same EVSE each day.

Figure 5 shows shows the daily operational costs for both the Lagrangian relaxation and one-shot heuristics across 100 days. The one-shot algorithm was implemented each day with $\epsilon = 0.10$. The Lagrangian relaxation performs poorly on early days due to exploration of prices. Once the algorithm finds the optimal prices, the Lagrangian relaxation solution produces total cost less than one-shot. Additionally, the one-shot algorithm does not suffer from the initial cost spike. This resulted in costs better than the Lagrangian relaxation until day 40; however, the one-shot solution never performs as well after this day. Clearly, if the incoming EV distribution was non-stationary, this conclusion would not be valid and the one-shot approach would be preferable.

VIII. Conclusion

In this paper, we proposed three posted price heuristics to maximize the smart charging potential of EVSEs in a parking structure. Our OCO solution provides fast price updates after each arrival and an average regret guarantee compared to the optimal offline solution. The Lagrangian relaxation solution updates EVSE prices once per day and converges to the optimal daily prices. Furthermore, the Lagrangian relaxation solution can be modified into a one-shot learning algorithm that uses the first fraction of arrivals to predict the daily prices allowing nonstationary inputs. The Lagrangian relaxation, one-shot, and OCO heuristics all provide posted prices for the arriving EVs to quickly solve their own assignment. In future work, we will include late arrivals and early departures into the system model. Additionally, we will further present the potential smart charging benefits (from implementations such as load shifting, demand response, and behind-the-meter renewable integration) for parking infrastructure utilizing smart assignment mechanisms.

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