Jump Linear Quadratic Control for Energy Management of a Nanogrid

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Off-The-Grid

- Increasing interest
  - “Tiny Houses”
  - Carbon footprint
- Developing areas
  - Power grid can be unreliable
  - Too costly to expand to remote areas
  - “Skipped” technology similar to cellular phones

Solar

- PV generation is abundant for households
- Clearly a positive, however...
- Systems solely equipped with solar have wasted potential
    - Can only generate during daytime
    - Dependent on power grid (night/bad weather)
    - Excess generation sent back to the grid

Solar + Battery Energy Storage Systems

- This combination facilitates several beneficial functions such as:
    - Solar self-consumption
    - Time of use load shifting
    - Backup power
    - Off the grid living
- Household equipped with both can be treated as its own microgrid system
- Only needs to use the connection to the existing utility network when absolutely necessary

Nanogrids

- Nanogrids are a new concept and are defined as microgrids under 5 kW
- In this work, we assume the nanogrid has no connection to the electric grid
- As such, control strategies must be used to ensure power balance
- In an isolated nanogrid, the control strategy:
  - does not need to handle multiple loads and control competition for a shared resource
  - relinquishes power generation and distribution responsibilities from the public grid
  - provides the consumer complete control

Nanogrid Architecture

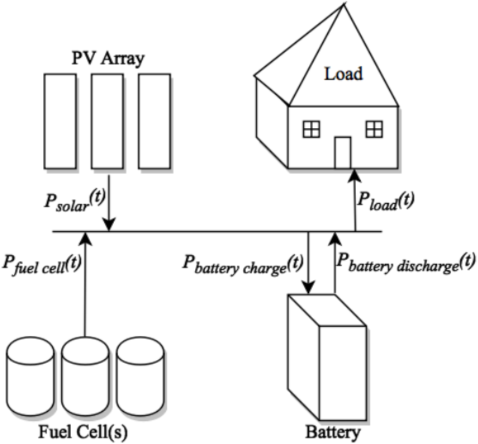
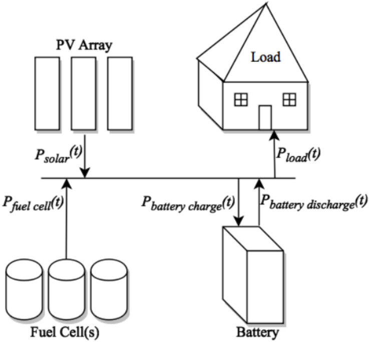


Figure: nGrid architecture with battery, fuel cell, solar, and load

Power Balance



- Power balance equation:

$$P_{load}(t) = P_{solar}(t) + P_{FC}(t) + P_{batt.}(t).$$

- Load model from NREL System Advisory Model (SAM)
- No connection to the local power grid (*isolated nanogrid*)

Solar Generation Model

- Solar generation has a random nature due to cloud coverage
- Markov chain to model stochastic cloud patterns
- Cloud coverage falls into one of three categories

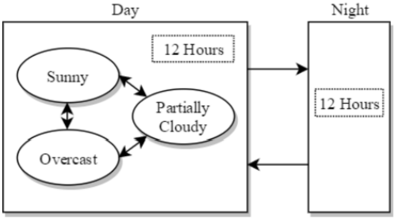


Figure: Weather diagram with a Markov chain representing cloud coverage during daytime

Solar Generation Model

- Power generated by solar:

$$P_{solar}(t) = \begin{cases} P_{solar}^{day}(t) = P_s(r(t)) = \begin{cases} r(t) = 1 & \text{(sunny)} \\ r(t) = 2 & \text{(cloudy)} \\ r(t) = 3 & \text{(overcast)} \end{cases} \\ P_{solar}^{night}(t) = 0 \end{cases}$$

- $r(t)$ evolves according to a continuous time Markov chain taking values in a finite set $S = \{1, 2, 3\}$ with transition matrix $\Pi = [\pi_{ij}]; i, j = 1, 2, 3$

Solar Generation Model

- Data for the cloud coverage transition probability matrix was gathered from San Jose International Airport's routine meteorological weather reports, also known as METAR
- Quantize the hourly cloud coverage from 2016 data into three levels: sunny, cloudy, and overcast:

$$\Pi = \begin{bmatrix} -0.197 & 0.164 & 0.033 \\ 0.085 & -0.179 & 0.094 \\ 0.010 & 0.138 & -0.148 \end{bmatrix}$$

Battery Energy Storage System

- Battery acts as a buffer, storing excess solar and fuel cell energy for later use
- Important specifications of the battery include energy capacity (more specifically SoC), charge/discharge powers, life cycle, as well as safe operating temperatures
- Energy limits: $\underline{E}_b \leq E_b(t) \leq \overline{E}_b$
- Charging and Discharging limits: $0 \leq P_{batt}^{ch.}(t) \leq \overline{P}_b^{ch.}$ and $0 \leq P_{batt}^{dis.}(t) \leq \overline{P}_b^{dis.}$
- It is common practice to set $\underline{E}_b \geq 10\%$ and $\overline{E}_b \leq 90\%$ of the maximum capacity to increase lifespan

Fuel Cell

- Fuel cells are effective generators for long periods of time, from several minutes up to several months
- Similar to the battery, the fuel cell has generation limits: $0 \leq P_{FC}(t) \leq \overline{P}_{FC}$
- In our model, we make the assumption that the total amount of energy the fuel cell is capable of supplying over time is not restricted

System Equations

- System may be described as a continuous linear time invariant system with Markovian jumps and a hybrid (continuous-discrete) state space $[x_1 \; x_2 \; r]^T$

$$\frac{dE_b(t)}{dt} = \alpha E_b(t) + (1 - a_i)P_{solar}(t) - b_i P_{batt}^{dis.}(t) + (1 - c_i)P_{FC}(t)$$

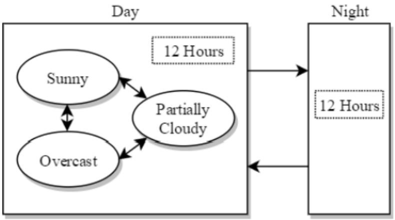
$$\frac{dE_l(t)}{dt} = \beta E_l(t) + a_i P_{solar}(t) + b_i P_{batt}^{dis.}(t) + c_i P_{FC}(t)$$

$$x(t) = \begin{bmatrix} E_b(t) \\ E_l(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} E_b(t) \\ E_l(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} P_{batt}^{dis}(t) \\ P_{FC}(t) \end{bmatrix},$$

$$P_{solar}(t) = P_s(r(t))$$

System Dynamics

- The system has unique state-space matrices, $[A_i, B_i]$, for each mode $r(t)$
- The system will operate in a continuous state with $r(t) = i$ until the next mode, $r(t) = j$, occurs
- During the night, $P_{solar}(t) = 0$ and the system switches to a deterministic structure



JLQ Control Applied to Nanogrid

- Objective is to find a control law that will supply the power required to meet the household energy demand
- We assume that cost of energy is the same for all sources
- We do away with the piecewise constant vector $P_{solar}(r(t))$ by means of the following change of variables:

$$\tilde{x}_1(t) = x_1(t) + (1 - a_i)P_i/\alpha$$

$$\tilde{x}_2(t) = x_2(t) + a_i P_i/\beta$$

JLQ Control Applied to Nanogrid

- At continuity points of $r(t)$:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B(r(t))\tilde{u}(t)$$

- At jump instants:

$$\tilde{x}(t) = \tilde{x}(t^-) + q_{ij}$$

$$q_{ij} = \begin{pmatrix} (1-a_i)P_i - (1-a_j)P_j \\ a_iP_i - a_jP_j \end{pmatrix}$$

- $\tilde{x}(t^-)$ is the limit from the left of $\tilde{x}(\tau)$ as τ goes to t and q_{ij} is the change due to constant perturbations that differ from mode i to mode j

JLQ Control - Steady State

- The state-space representation of a jump linear system

$$\dot{x}(t) = A\big(r(t)\big)x(t) + B\big(r(t)\big)u(t)$$

- Steady state: the Riccati gains $K_i(t)$ for the infinite horizon problem will converge to K_i^{∞}

$$A_i^T K_i^{\infty} + K_i^{\infty} A_i + Q_i - K_i^{\infty} S_i K_i^{\infty} + \sum_{j=1}^{N} \pi_{ij} K_j^{\infty} = 0$$

- The resulting control law becomes:

$$u^*(t) = -R_i^{-1} B_i^T K_i^{\infty} x(t) \quad \text{for} \quad r(t) = i$$

JLQ Control Applied to Nanogrid

- Time varying feedback law affine in state:

$$u^*(t) = -R_i^{-1}B_i^T K_i^{\infty}(\tilde{x}(t) + \alpha_i) \quad \text{for} \quad r(t) = i$$

- Where the bias vector $\alpha_i(t)$ evolves:

$$\dot{\alpha}_i(t) = (A + K_i^{-1}Q_i)\alpha_i(t) + \sum_{j=1}^{3} \pi_{ij} K_i^{-1}K_j(\alpha_i(t) - \alpha_j(t) - q_{ij})$$

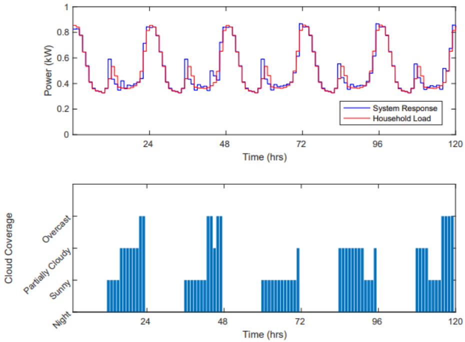
$$\alpha_i(t_f) = 0 \quad i = 1,2,3$$

Example System Specifications

- The load pertains to a small U.S. household, one similar in size and energy usage to common off-the-grid homes
- We employed a 7kWh battery with constraints: $\underline{E}_b = 1$kWh and $\overline{E}_b = 6$kWh
- During a sunny day, the solar production completely satisfies the demand with some excess energy that can be stored in the battery for later use
- The fuel cell was sized to satisfy the demand when the battery and solar could not

5 Day Response

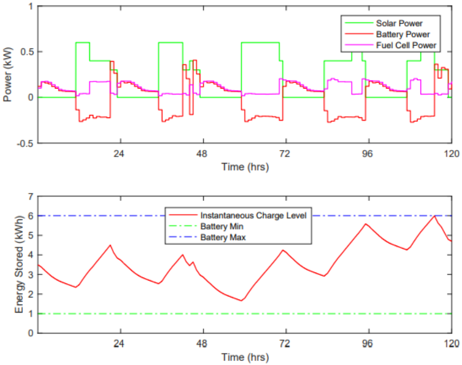
System Response and Cloud Coverage



- Load with simulated system response (top graph)
- Simulated cloud coverage (bottom graph)
- Load model from NREL System Advisory Model (SAM)

5 Day Response

Control Actions and Battery Energy Level



- Control actions (top graph)
- Energy stored in battery (bottom graph)
- Due to the mostly sunny weather of this 5-day simulation, the battery never reaches the minimum allowed charge level

Conclusion

- In this talk:
    - JLQ control applied to the energy management of a nanogrid
    - Solar generation modeled as a continuous Markov process
    - Simulation results showed the proposed control method was able to track a home's load profile
- Future work:
    - More accurate battery and fuel cell models
    - Look into more effective ways to handle jump instants
    - First passage of time theory for battery SOC management