

# An Online Pricing Mechanism for Electric Vehicle Parking Assignment and Charge Scheduling

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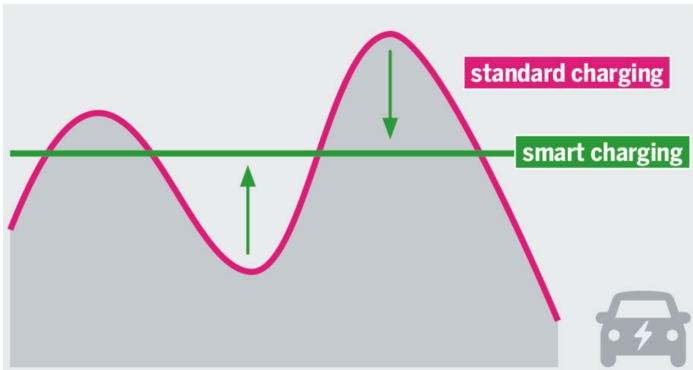
American Control Conference, 2019, Philadelphia

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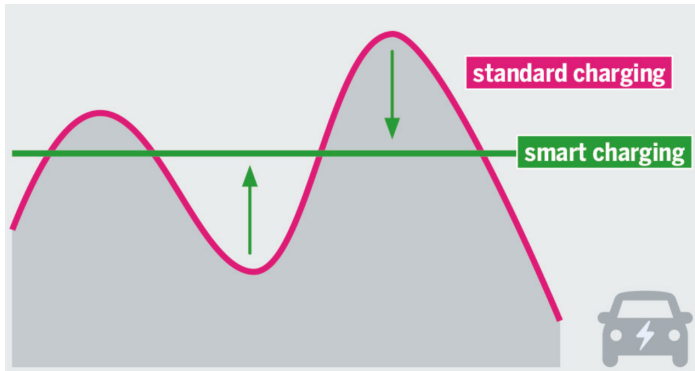
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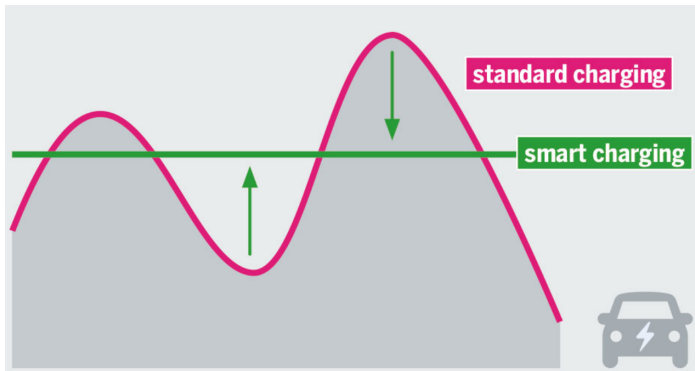
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- Cannot fully integrate **renewable** power generation

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- Public parking facilities have unused **smart charging** potential
- Can we utilize existing **smart charging** methods for public parking facilities equipped with chargers?

**\*Unfortunately, no\***

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**\*Require **online** management systems for admission decisions and **shared resource** allocation to enable **smart charging**\***

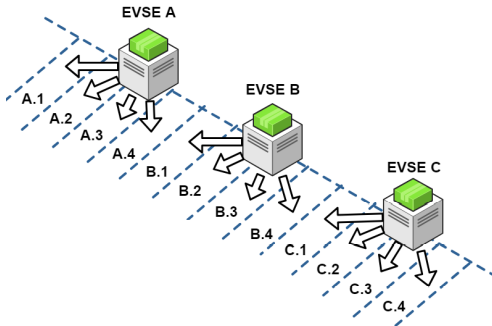
## Goal

Design **online** reservation and pricing strategies for **public** facilities equipped with **shared** EV chargers to enable **smart charging**



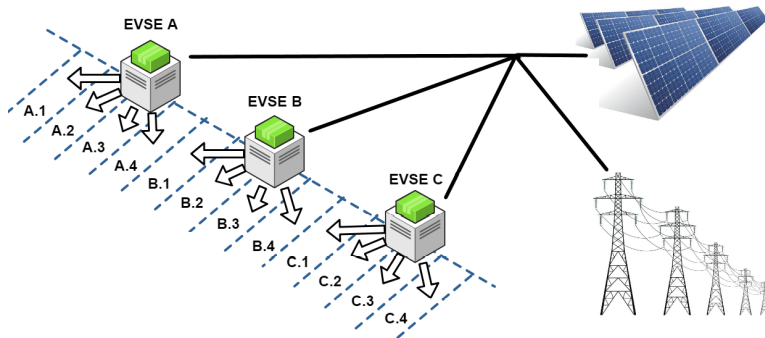
# System Description

- $L$  dispersed parking facilities with multiple-cable chargers



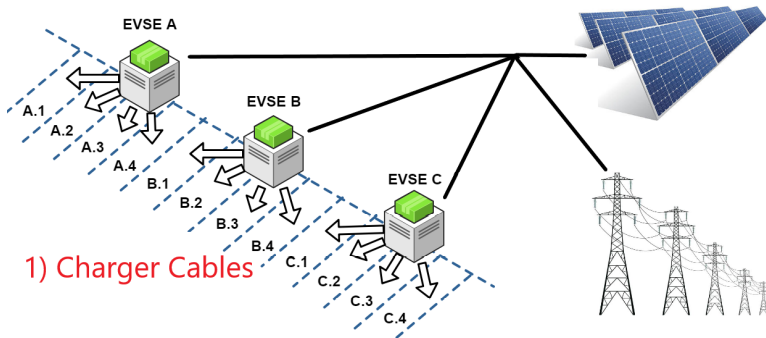
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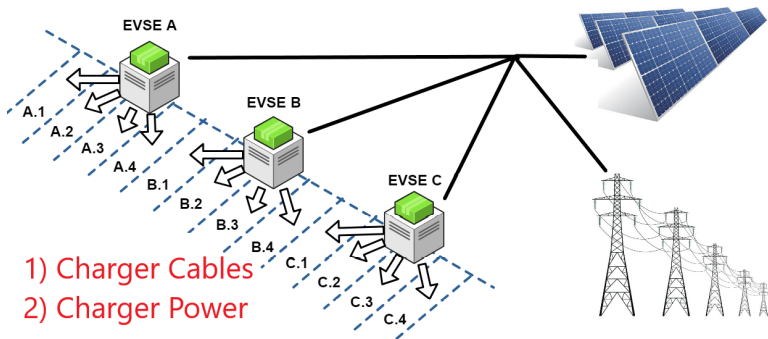
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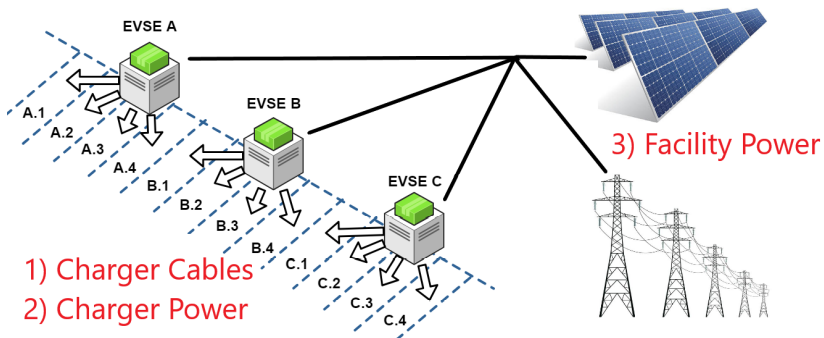
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$$\theta_n = \{t_n^-, t_n^+, h_n, \{\ell_n\}, \{v_{n\ell}\}\} \in \Theta$$

- $t_n^-$ : User  $n$ 's arrival time
- $t_n^+$ : User  $n$ 's departure time
- $h_n$ : User  $n$ 's desired energy amount
- $\{\ell_n\}$ : User  $n$ 's preferred facilities
- $\{v_{n\ell}\}$ : User  $n$ 's valuations for charging at each facility  $\ell$



# Example Reservation Schedule

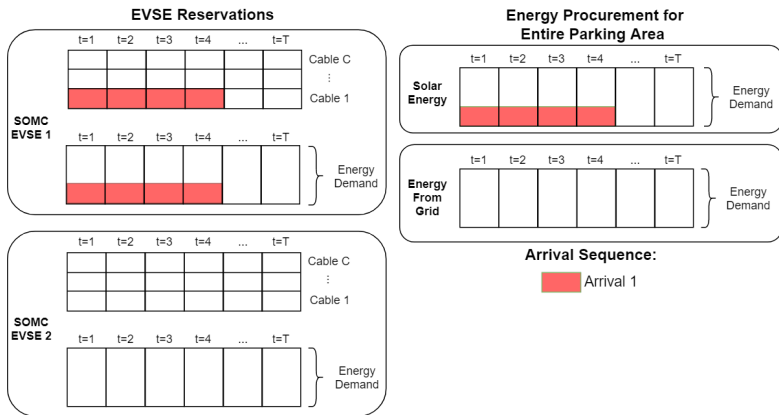


Figure: Facility schedule after 1 arrival.

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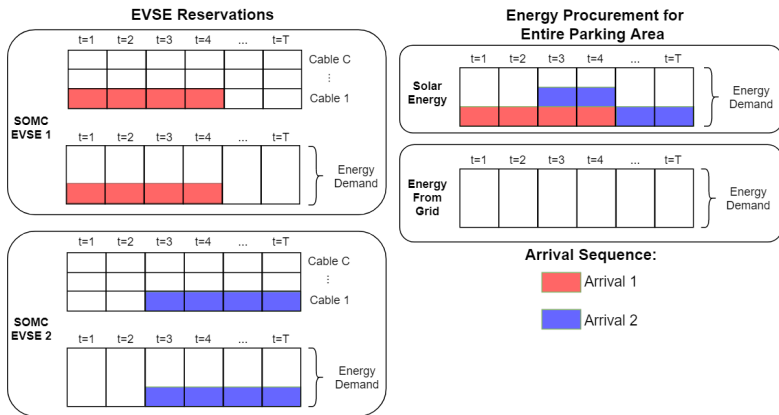


Figure: Facility schedule after 2 arrivals.

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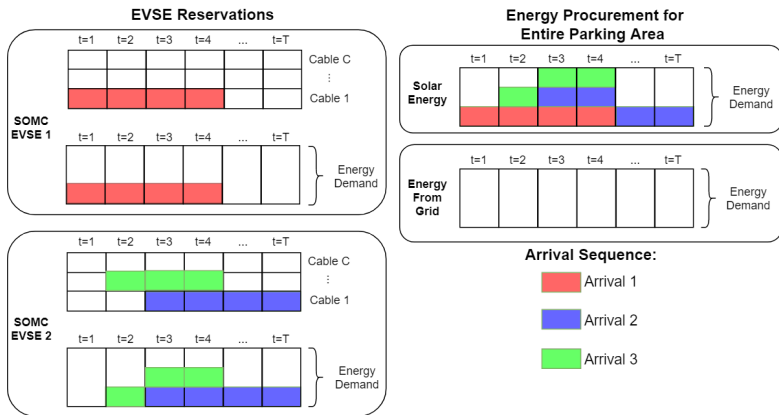


Figure: Facility schedule after 3 arrivals.

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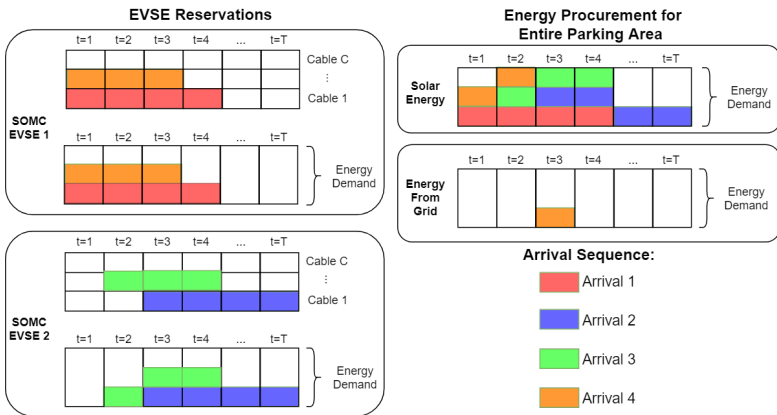


Figure: Facility schedule after 4 arrivals.

# Offline Social Welfare Maximization Problem

$$\max_x \sum_{\mathcal{N}, \mathcal{O}_n, \mathcal{L}, \mathcal{M}_\ell} v_{nl} x_{no}^{ml} - \sum_{\mathcal{T}, \mathcal{L}} f_g^\ell(y_g^\ell(t))$$

subject to:

$$\sum_{\mathcal{O}_n, \mathcal{L}, \mathcal{M}_\ell} x_{no}^{ml} \leq 1, \quad \forall n$$

$$x_{no}^{ml} \in \{0, 1\}, \quad \forall n, o, l, m$$

$$y_c^{ml}(t) \leq C_\ell, \quad \forall l, m, t$$

$$y_e^{ml}(t) \leq E_\ell, \quad \forall l, m, t$$

## Facilities' Electricity Costs

The energy procurement,  $y_g^\ell(t)$ , determines the operational cost of facility  $\ell$  (i.e., purchasing electricity from the distribution grid):

$$f_g^\ell(y_g^\ell(t)) = \begin{cases} 0 & y_g^\ell(t) \in [0, s_\ell(t)) \\ \pi_\ell(t)(y_g^\ell(t) - s_\ell(t)) & y_g^\ell(t) \in [s_\ell(t), s_\ell(t) + G_\ell(t)] \\ +\infty & y_g^\ell(t) > s_\ell(t) + G_\ell(t) \end{cases}$$

## Admittance, Rejection, and Allocation Decisions

- Can examine KKT conditions for the dual constraints:

$$u_n \geq 0$$

$$u_n \geq v_{nl} - \sum_{\mathcal{T}} \left( c_{no}^{m\ell}(t) p_c^{m\ell}(t) + e_{no}^{m\ell}(t) (p_e^{m\ell}(t) + p_g^\ell(t)) \right)$$

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$$u_n = \max \left\{ 0, \max_{\mathcal{O}_n, \mathcal{L}, \mathcal{M}_l} \left\{ v_{nl} - \sum_{t \in [t_n^-, t_n^+]} \left( c_{no}^{ml}(t) p_c^{ml}(t) + e_{no}^{ml}(t) (p_e^{ml}(t) + p_g^l(t)) \right) \right\} \right\}$$



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- Provide **performance guarantees**



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$$p_g^\ell(y_g^\ell(t)) = \begin{cases} \left( \frac{L_g}{2R} \right) \left( \frac{2R\pi_\ell(t)}{L_g} \right)^{\frac{y_g^\ell(t)}{s_\ell(t)}} & y_g^\ell(t) < s_\ell(t) \\ \left( \frac{L_g - \pi_\ell(t)}{2R} \right) \left( \frac{2R(U_g - \pi_\ell(t))}{L_g - \pi_\ell(t)} \right)^{\frac{y_g^\ell(t)}{s_\ell(t) + G_\ell(t)}} + \pi_\ell(t) & y_g^\ell(t) \geq s_\ell(t) \end{cases}$$

# Performance Guarantee: Competitive Ratio

- Competitive ratio:

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- An online mechanism is “ $\alpha$ -competitive” when:

$$\alpha \geq \frac{\text{Optimal Offline Solution's Social Welfare}}{\text{Worst Case[Online Mechanism's Social Welfare]}} \geq 1$$

# Online Reservation System Competitive Ratio

The online EV charger reservation system that makes use of our heuristic price update functions is  $\alpha_1$ -competitive in social welfare where

$$\alpha_1 = 2 \max_{\mathcal{L}, \mathcal{T}} \left\{ \ln \left( \frac{2R(U_g - \pi_\ell(t))}{L_g - \pi_\ell(t)} \right) \right\}.$$

## Competitive Ratio: Imperfect Solar Forecast

- Daily solar generation forecast as a confidence interval:

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Using the lower bound solar forecast, the reservation system is  $\alpha_2$ -competitive in social welfare where

$$\alpha_2 = 2 \max_{\mathcal{L}, \mathcal{T}} \left\{ \left( \frac{\bar{s}_\ell(t) + G_\ell(t)}{\underline{s}_\ell(t) + G_\ell(t)} \right) \ln \left( \frac{2R(U_g - \pi_\ell(t))}{L_g - \pi_\ell(t)} \right) \right\}.$$

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$$(p(t) - f'(y(t)))dy(t) \geq \frac{1}{\alpha(t)} f^{*'}(p(t))dp(t)$$

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- Resulting  $\alpha_1$  is the maximum  $\alpha(t)$  over all facilities, resources, and time.

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## Comparison with First-Come-First-Serve

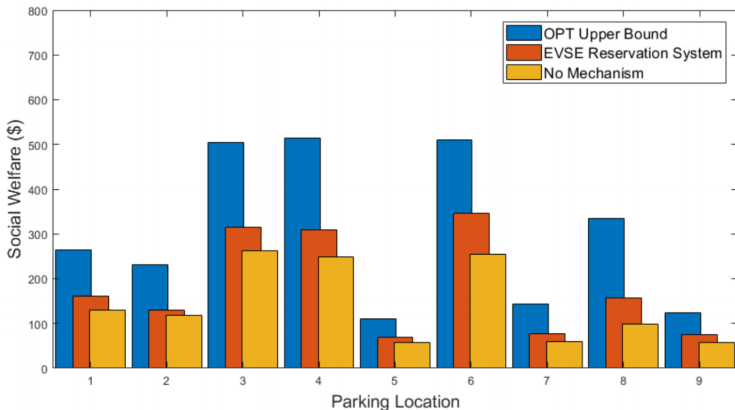


Figure: Social welfare for 9 downtown parking facilities

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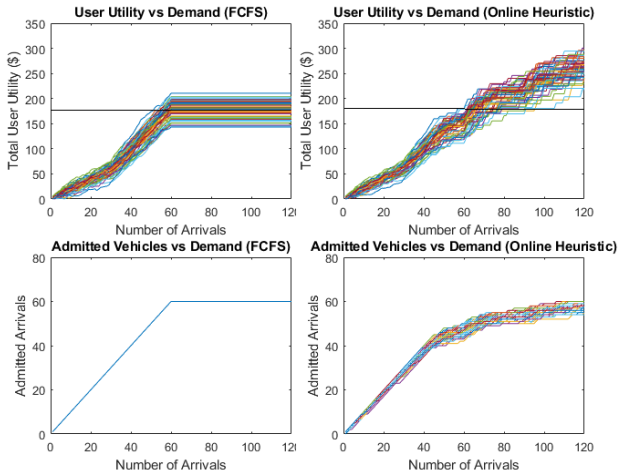


Figure: Left: FCFS. Right: Online Mechanism

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Online reservation system for public parking facilities via heuristic pricing functions in order to enable smart charging:

1. Admission controller for public parking facility access
2. Shared resource manager that optimizes smart charging strategies for vehicles admitted to the facilities
3. Able to account for stochastic renewable generation
4. Robust to adversarially chosen arrival sequences and is  $\alpha$ -competitive in social welfare to the optimal offline solution