Online Optimization and Learning for Sustainable Human-Cyber-Physical Systems

Nathaniel Tucker

Wednesday, November 18, 2020

Global Energy Transition

Two Major Components

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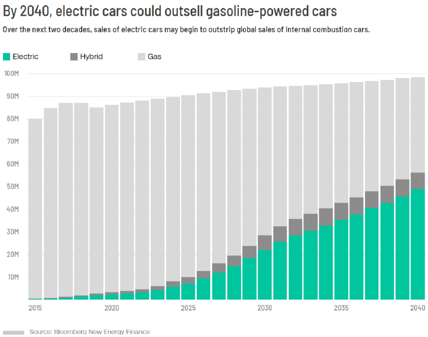
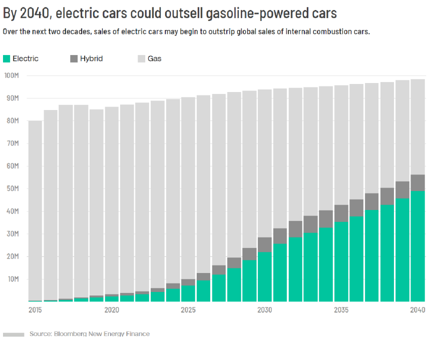


Transportation Electrification

Global Energy Transition

Two Major Components



Transportation Electrification

- Infrastructure management
- Effects on the grid

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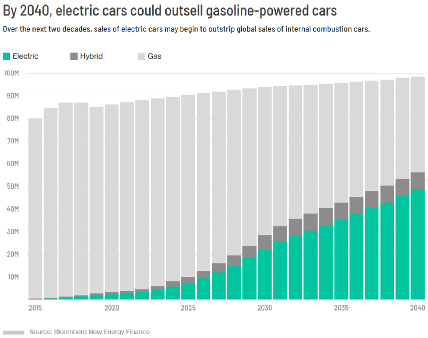

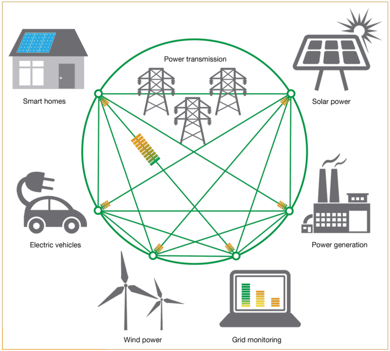

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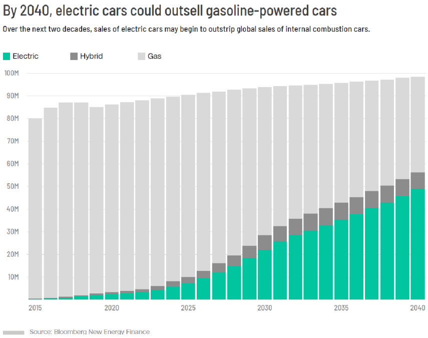

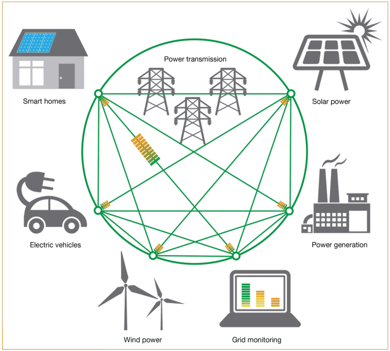

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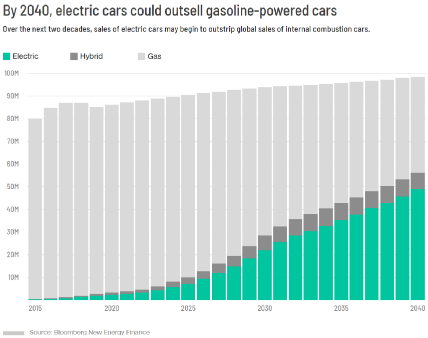

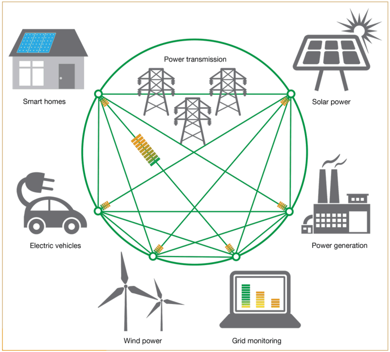

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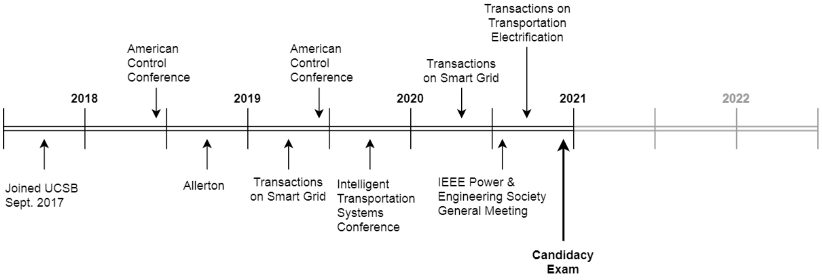
Both can benefit from optimization and learning mechanisms

Timeline



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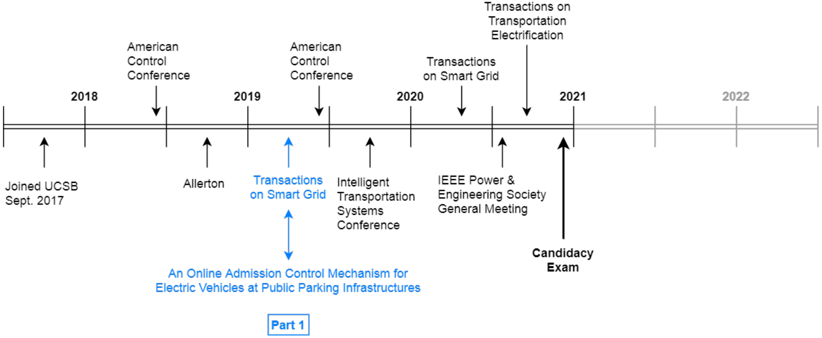

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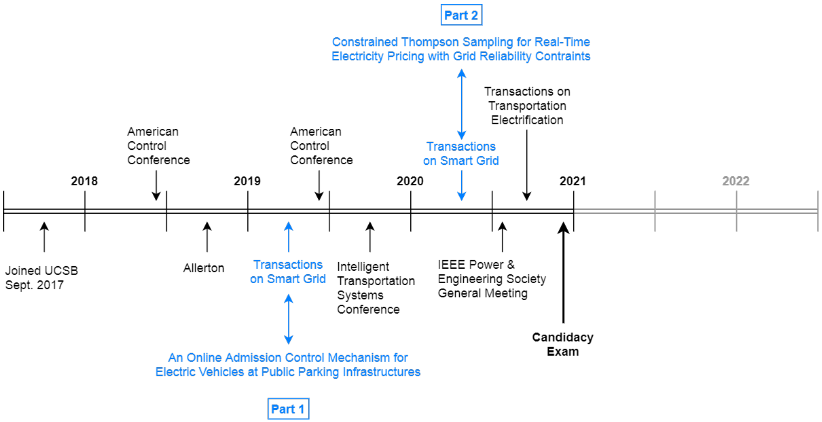
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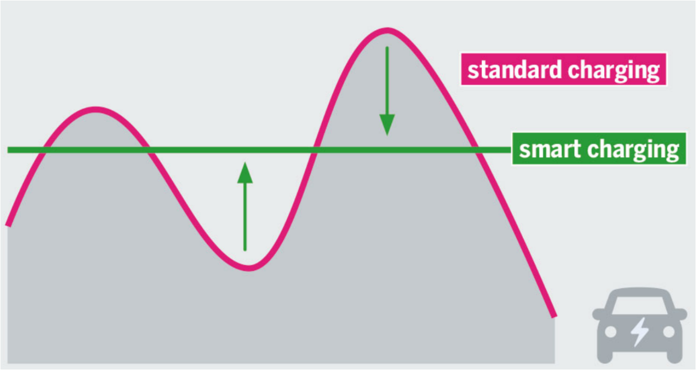

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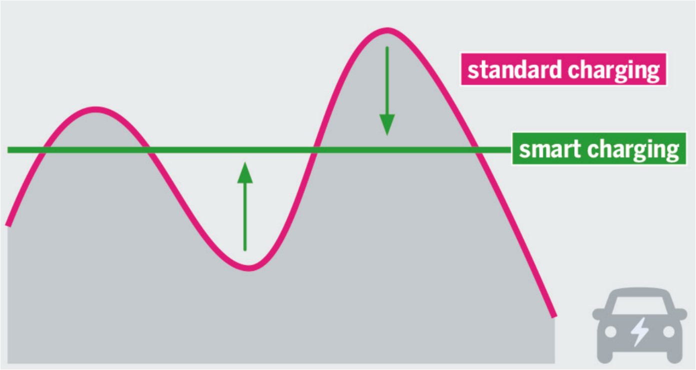

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- Resulting power demand could negatively affect the grid (i.e., high demand during peak hours)

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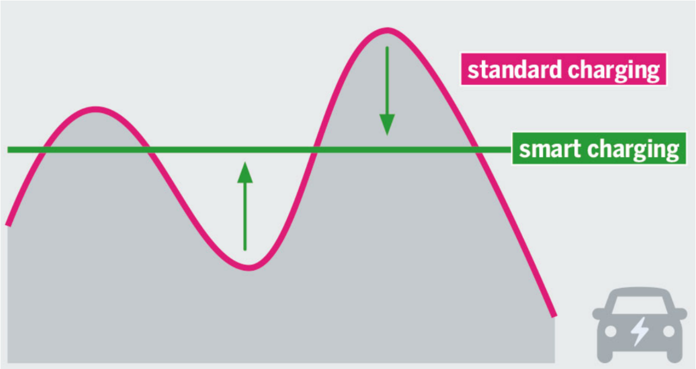

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- Cannot fully integrate renewable power generation

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- Can we utilize existing smart charging methods for public parking facilities equipped with chargers?

Unfortunately, no

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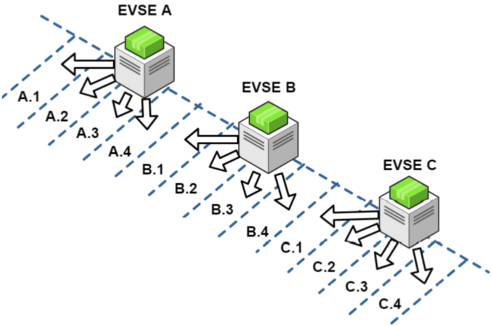
Require online management systems for admission decisions and shared resource allocation to enable smart charging

System Description

- L dispersed parking facilities with multiple-cable chargers



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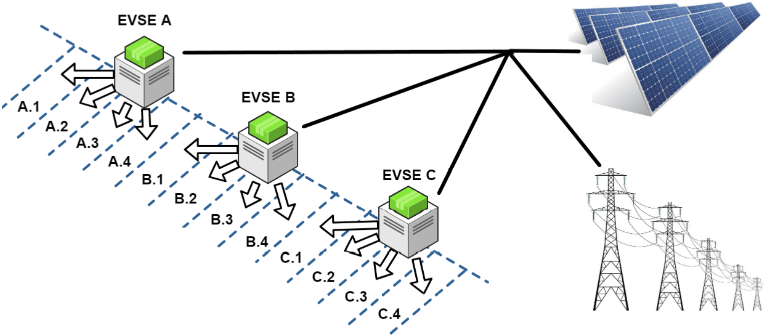

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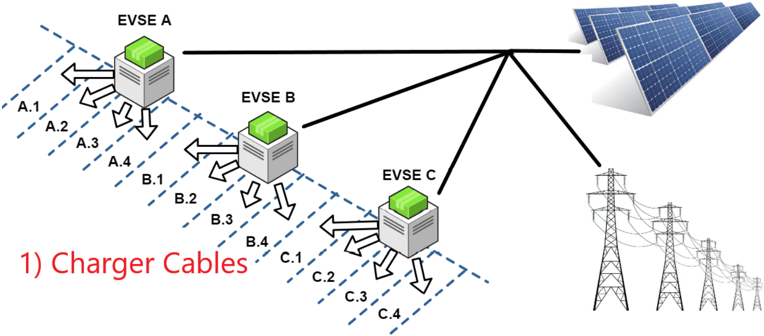

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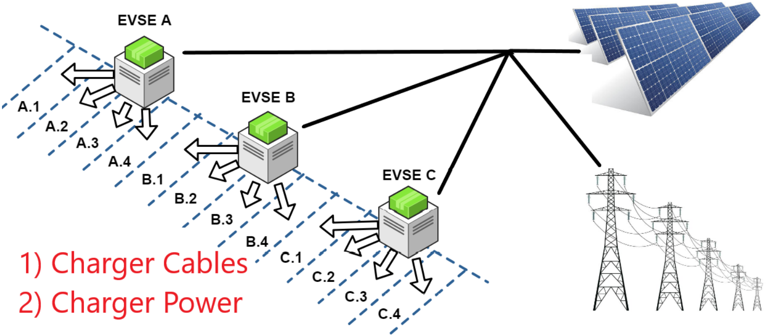

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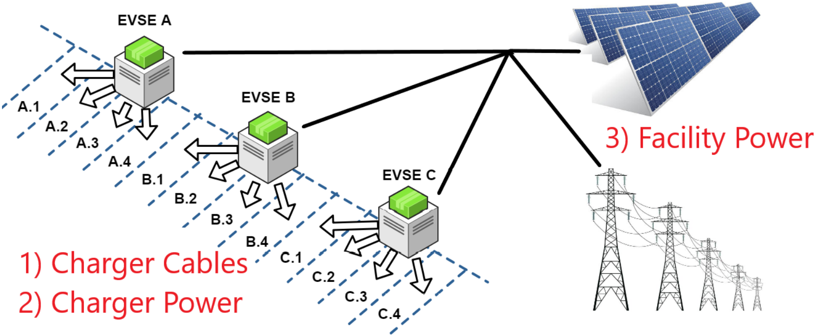

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- t_n^-: User n's arrival time
- t_n^+: User n's departure time
- h_n: User n's desired energy amount
- $\{\ell_n\}$: User n's preferred facilities
- $\{v_{n\ell}\}$: User n's valuations for charging at each facility ℓ

Parking and Charging Reservation *Options*

- There are a set of options $\mathcal{O}_n$ that fulfill user n's type (θ_n):

$$\{t_n^-, t_n^+, \{c_{no}^{m\ell}(t)\}, \{e_{no}^{m\ell}(t)\}, \{\ell_n\}, \{v_{n\ell}\}\}$$

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- $c_{no}^{m\ell}(t)$: Binary cable reservation; 1 if user n is assigned a cable from EVSE m at facility ℓ at time t in option o; 0 otherwise
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- If there were posted prices for these options, users could select their utility maximizing reservation

Example Reservation Schedule

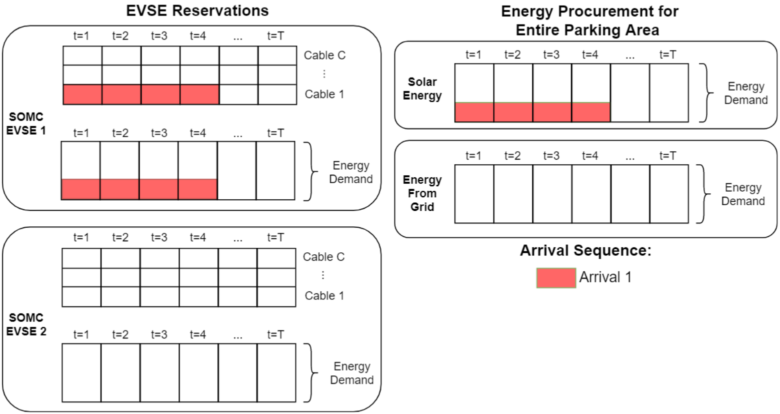

Figure: Facility schedule after 1 arrival.

Example Reservation Schedule

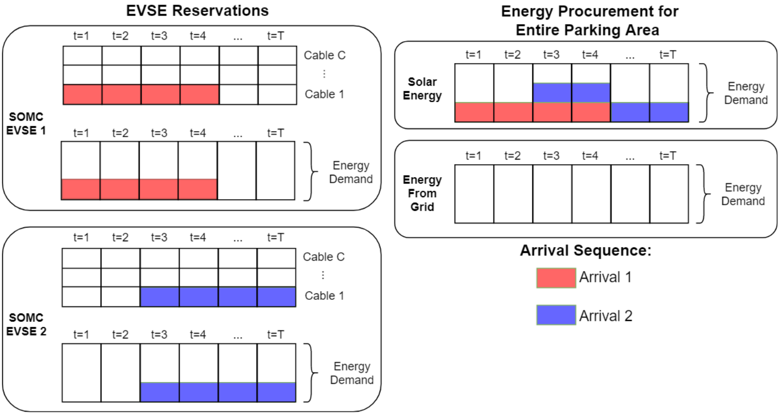

Figure: Facility schedule after 2 arrivals.

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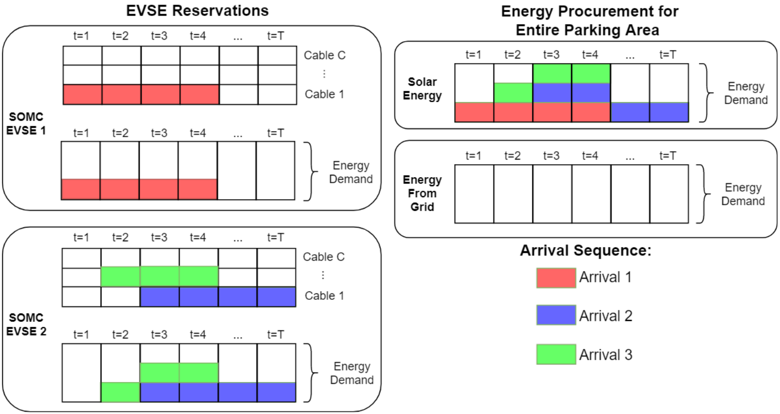

Figure: Facility schedule after 3 arrivals.

Example Reservation Schedule

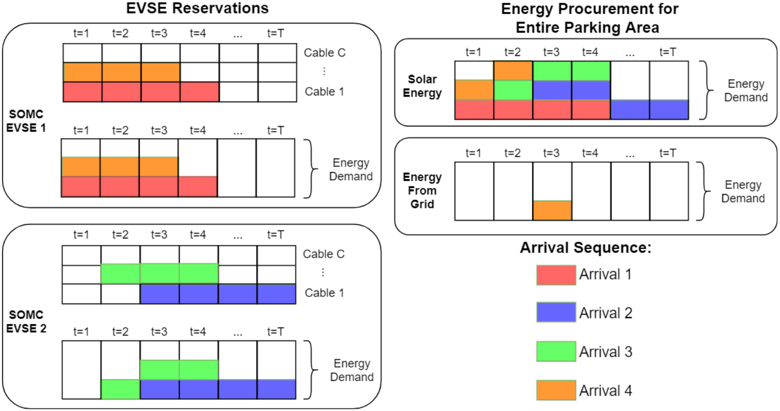

Figure: Facility schedule after 4 arrivals.

Offline Social Welfare Maximization Problem

$$\max_{x} \sum_{\mathcal{N},\mathcal{O}_n,\mathcal{L},\mathcal{M}_\ell} v_{n\ell} x_{no}^{m\ell} - \sum_{\mathcal{T},\mathcal{L}} f_g^\ell(y_g^\ell(t))$$

subject to:

$$\sum_{\mathcal{O}_n,\mathcal{L},\mathcal{M}_\ell} x_{no}^{m\ell} \leq 1, \quad \forall\ n$$

$$x_{no}^{m\ell} \in \{0,1\}, \quad \forall\ n, o, \ell, m$$

$$y_c^{m\ell}(t) \leq C_\ell, \quad \forall\ \ell, m, t$$

$$y_e^{m\ell}(t) \leq E_\ell, \quad \forall\ \ell, m, t$$

Facilities' Electricity Costs

The energy procurement, $y_g^\ell(t)$, determines the operational cost of facility ℓ (i.e., purchasing electricity from the distribution grid):

$$f_g^\ell(y_g^\ell(t)) = \begin{cases} 0 & y_g^\ell(t) \in [0, s_\ell(t)\big) \\ \pi_\ell(t)(y_g^\ell(t) - s_\ell(t)) & y_g^\ell(t) \in [s_\ell(t), s_\ell(t) + G_\ell(t)] \\ +\infty & y_g^\ell(t) > s_\ell(t) + G_\ell(t) \end{cases}$$

How to select parking and charging reservations?

- Can examine the dual constraints:

$$u_n \geq 0$$

$$u_n \geq \; v_{n\ell} - \sum_{\mathcal{T}} \Big(c_{no}^{m\ell}(t) p_c^{m\ell}(t) + e_{no}^{m\ell}(t) \big(p_e^{m\ell}(t) + p_g^{\ell}(t) \big) \Big)$$

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$$u_n = \max \Big\{ 0, \max_{\mathcal{O}_n, \mathcal{L}, \mathcal{M}_\ell} \big\{ v_{n\ell} - \sum_{t \in [t_n^-, t_n^+]} \big(c_{no}^{m\ell}(t) p_c^{m\ell}(t) + e_{no}^{m\ell}(t) (p_e^{m\ell}(t) + p_g^{\ell}(t)) \big) \big\} \Big\}$$

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- If $u_n > 0$, user n purchases their utility maximizing parking and charging reservation, and is charged the following cost:

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- Post reservation prices, users select to maximize own utility
- Payment at the time of admission
- Handle adversarial arrival sequences (due to the unpredictable arrival distributions)
- Provide performance guarantees

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$$p_g^{\ell}(y_g^{\ell}(t)) =$$

$$\begin{cases} \left(\frac{L_g}{2R}\right)\left(\frac{2R\pi_\ell(t)}{L_g}\right)^{\frac{y_g^\ell(t)}{s_\ell(t)}} & y_g^\ell(t) < s_\ell(t) \\ \left(\frac{L_g-\pi_\ell(t)}{2R}\right)\left(\frac{2R(U_g-\pi_\ell(t))}{L_g-\pi_\ell(t)}\right)^{\frac{y_g^\ell(t)}{s_\ell(t)+G_\ell(t)}} + \pi_\ell(t) & y_g^\ell(t) \geq s_\ell(t) \end{cases}$$

Performance Guarantee: Competitive Ratio

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$$\frac{\text{Optimal Offline Solution's Social Welfare}}{\text{Worst Case[Online Mechanism's Social Welfare]}} \geq 1$$

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- Competitive ratio:

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- An online mechanism is "α-competitive" when:

$$\alpha \geq \frac{\text{Optimal Offline Solution's Social Welfare}}{\text{Worst Case[Online Mechanism's Social Welfare]}} \geq 1$$

Online Reservation System Competitive Ratio

The online EV charger reservation system that makes use of our heuristic price update functions is α_1-competitive in social welfare where

$$\alpha_1 = 2 \max_{\mathcal{L},\mathcal{T}} \left\{ \ln \left(\frac{2R(U_g - \pi_\ell(t))}{L_g - \pi_\ell(t)} \right) \right\}.$$

Competitive Ratio: Imperfect Solar Forecast

- Daily solar generation forecast as a confidence interval:

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$$f_g^\ell(y_g^\ell(t)) = \begin{cases} 0 & y_g^\ell(t) \in [0, s_\ell(t)) \\ \pi_\ell(t)(y_g^\ell(t) - s_\ell(t)) & y_g^\ell(t) \in [s_\ell(t), s_\ell(t) + G_\ell(t)] \\ +\infty & y_g^\ell(t) > s_\ell(t) + G_\ell(t) \end{cases}$$

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Using the lower bound solar forecast, the reservation system is α_2-competitive in social welfare where

$$\alpha_2 = 2 \max_{\mathcal{L},\mathcal{T}} \left\{ \left(\frac{\overline{s}_\ell(t) + G_\ell(t)}{\underline{s}_\ell(t) + G_\ell(t)} \right) \ln \left(\frac{2R(U_g - \pi_\ell(t))}{L_g - \pi_\ell(t)} \right) \right\}.$$

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- Resulting competitive ratio is the maximum $\alpha(t)$ over all facilities, resources, and time.

Comparison with First-Come-First-Serve

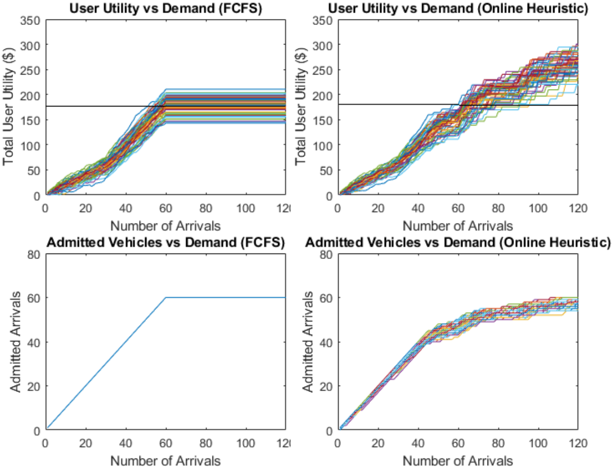

Figure: Left: FCFS. Right: Online Mechanism

Conclusion

Online reservation system for public parking facilities via heuristic pricing functions in order to enable smart charging:

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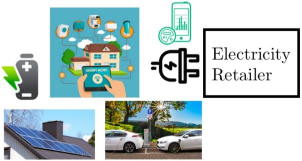
Online reservation system for public parking facilities via heuristic pricing functions in order to enable smart charging:

1. Admission controller for public parking facility access
2. Shared resource manager that optimizes smart charging strategies for vehicles admitted to the facilities
3. Able to account for stochastic renewable generation
4. Robust to adversarially chosen arrival sequences and is α-competitive in social welfare to the optimal offline solution

Part 2

Constrained Thompson Sampling for Real-Time Electricity Pricing with Grid Reliability Constraints

Demand Side Management



Demand side management is an increasingly popular control action that can be used to match consumption and generation

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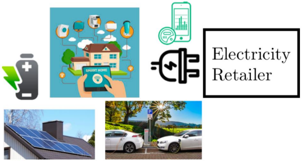

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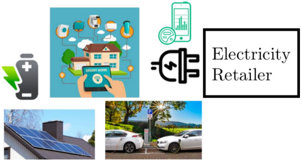

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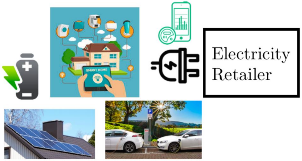

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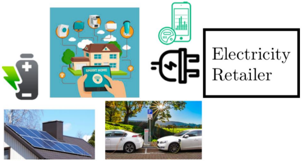

Demand side management is an increasingly popular control action that can be used to match consumption and generation

- Distributed coordination algorithms to load shape exist
- Often complicated, require 2-way communication
- Pricing + observing is a simpler framework
- Can we propose a smarter approach within this framework?

System Description

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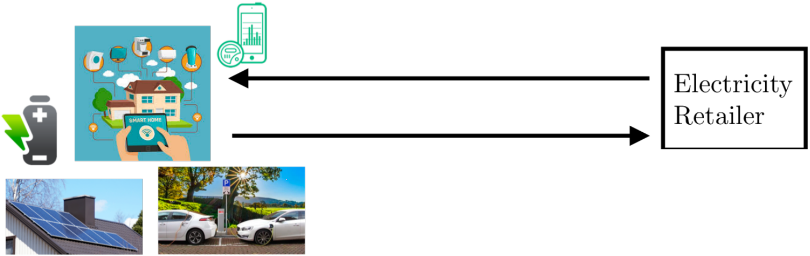

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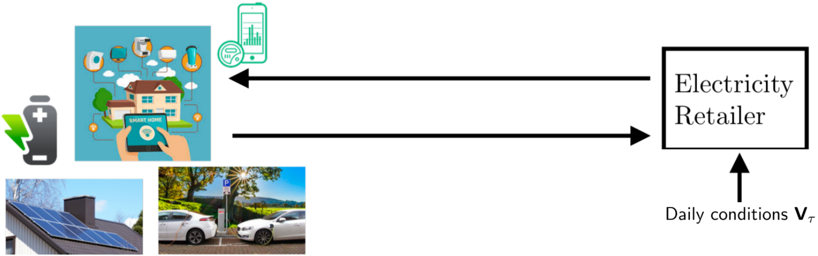

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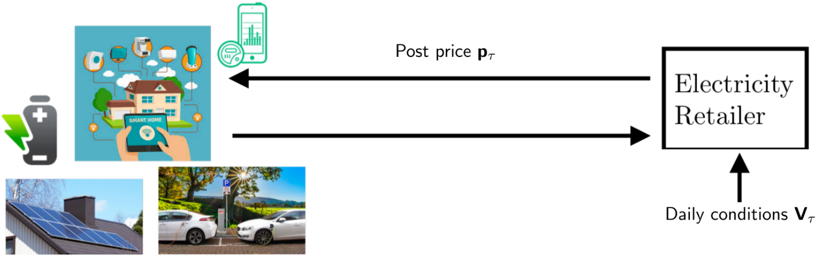

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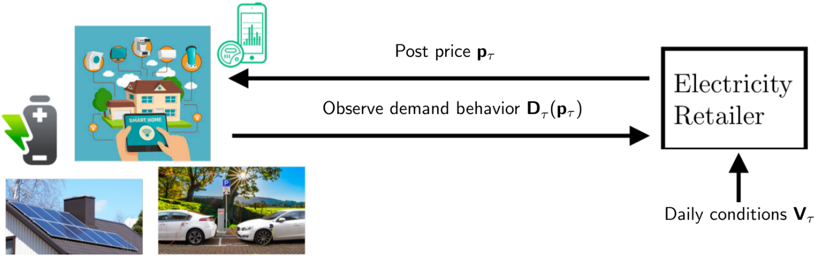

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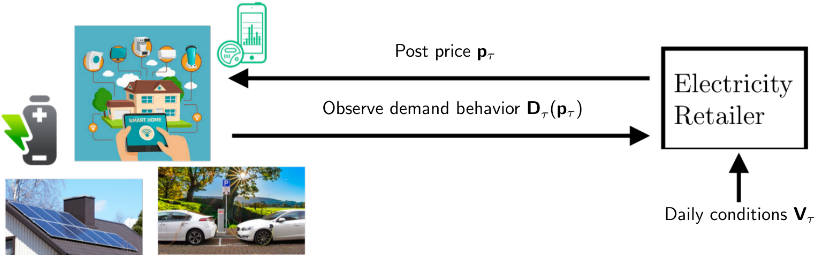

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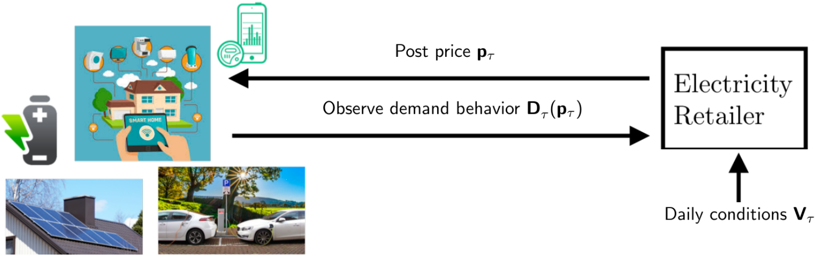

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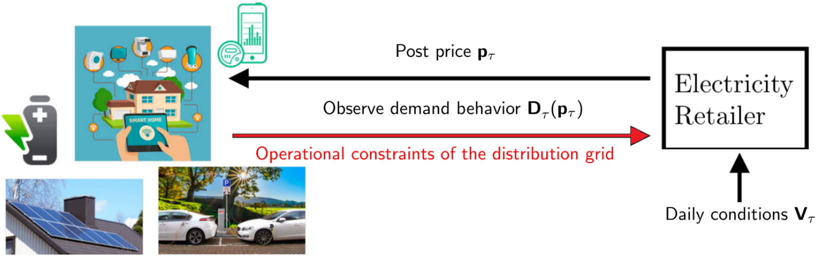

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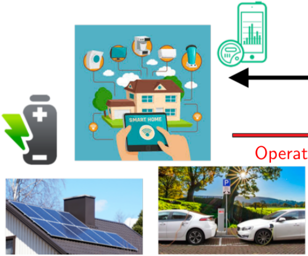

Post price $\mathbf{p}_\tau$

Observe demand behavior $\mathbf{D}_\tau(\mathbf{p}_\tau)$

Operational constraints of the distribution grid

Electricity Retailer

Daily conditions $\mathbf{V}_\tau$

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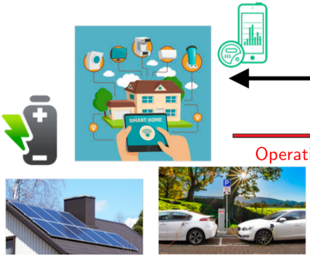

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Load Flexibility Model

What can the aggregator do to simplify learning a population's load response $\mathbf{D}_\tau(\mathbf{p}_\tau)$?

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What can the aggregator do to simplify learning a population's load response $\mathbf{D}_\tau(\mathbf{p}_\tau)$?

- Flexible loads only show a limited number of "load signatures" and can be clustered
- Due to automation, each flexible load selects its cost minimizing profile
- Uncertainty in $\mathbf{D}_\tau(\mathbf{p}_\tau)$ is reduced to the uncertainty of the number of appliances in each cluster
- Denote the number of flexible appliances in cluster c as $a_c(\mathbf{p}_\tau)$

Stochastic Customer Response

- Random or exogenous parameters lead to variability in temporal and geographical behavior

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- $\boldsymbol{\theta}^\star$ represents the *true model* for the customers' sensitivity to the price signals

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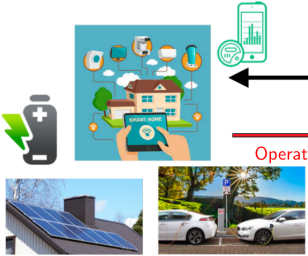

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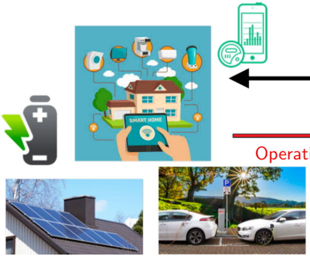

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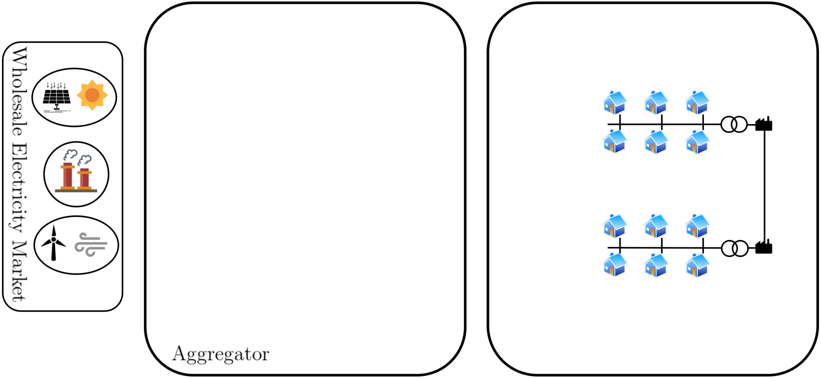

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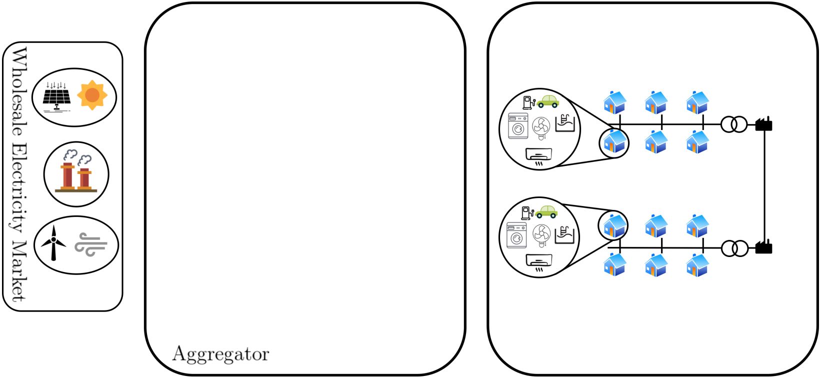

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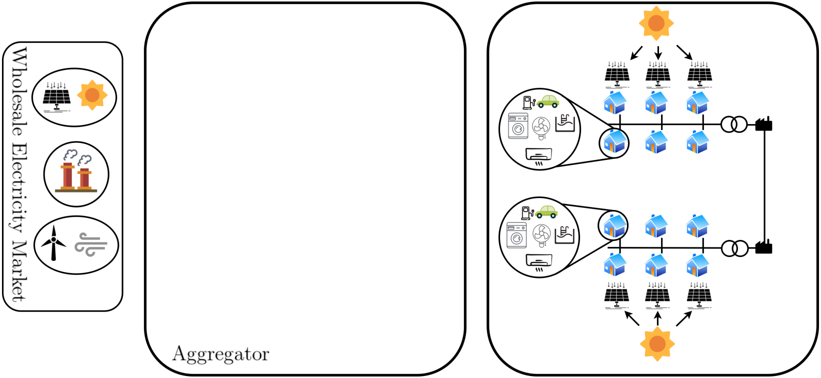

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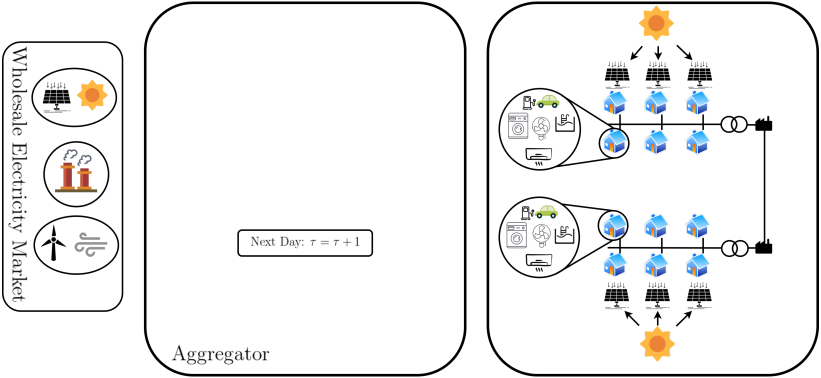

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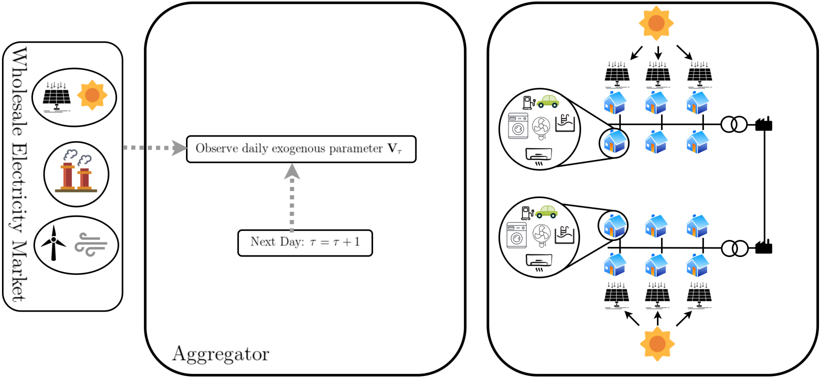

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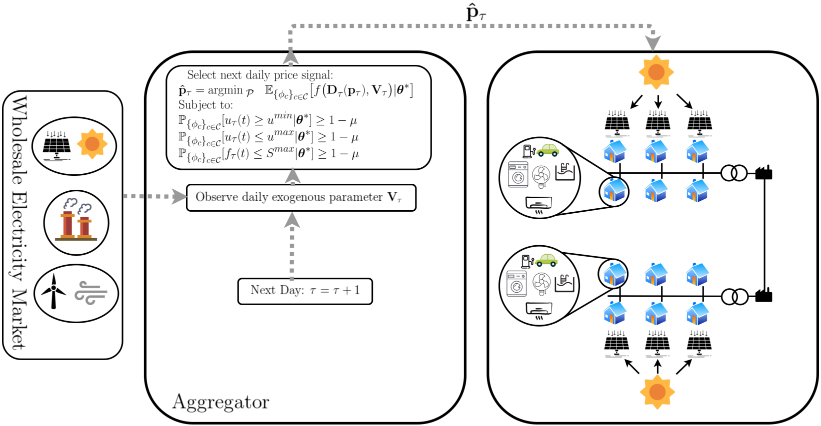

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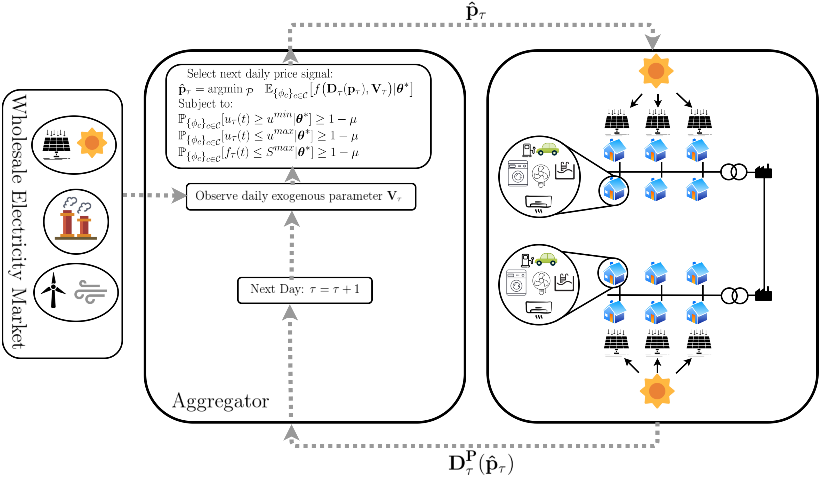

Multi-Armed Bandit

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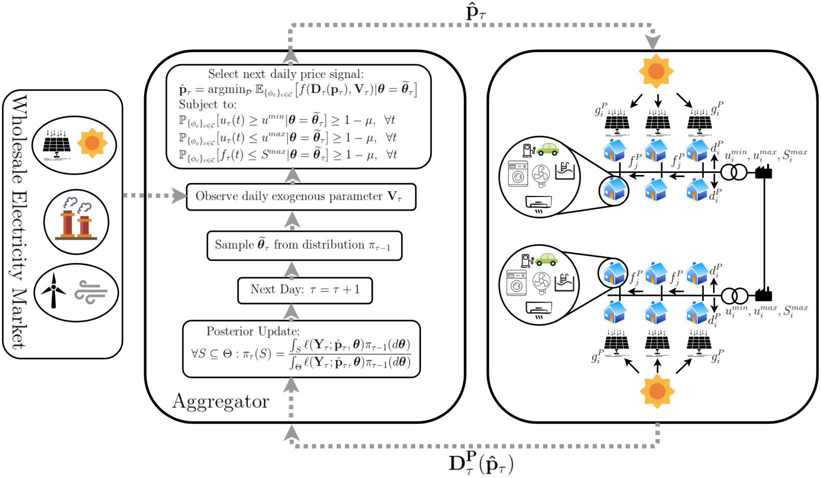
- Aggregator can only learn the consumers' responses ($\boldsymbol{\theta}^\star$) by experimenting with different price signals
- *Exploration vs. Exploitation* trade-off
- Goal is to develop a strategy for selecting price signals that balances this trade-off and minimizes the cumulative cost over a given time span

Constrained Thompson Sampling (Con-TS-RTP)



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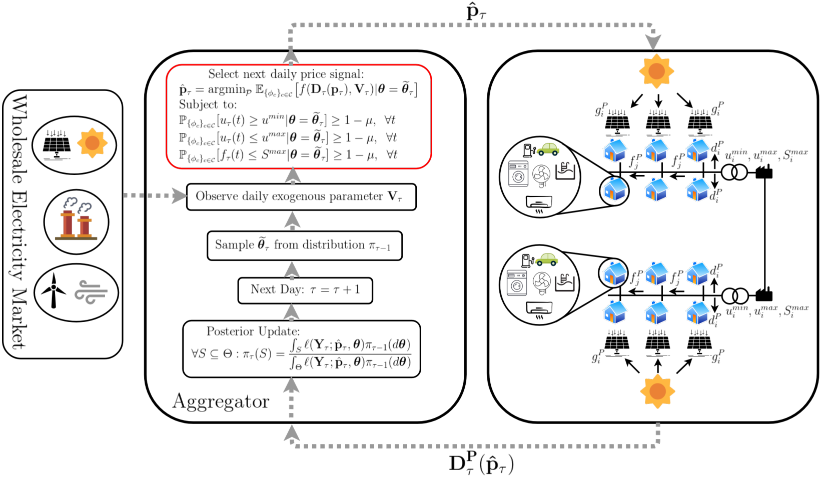
- Assumption 1: Finitely many price signals
- Assumption 2: Finite prior, *grain of truth*
- Assumption 3: Unique optimal price signal
- Under assumptions 1-3, Gopalan, et al. [1] proved that the number of suboptimal actions can be bounded and Moradipari, et al. [2] extended this result to account for exogenous parameters, $\mathbf{V}_\tau$

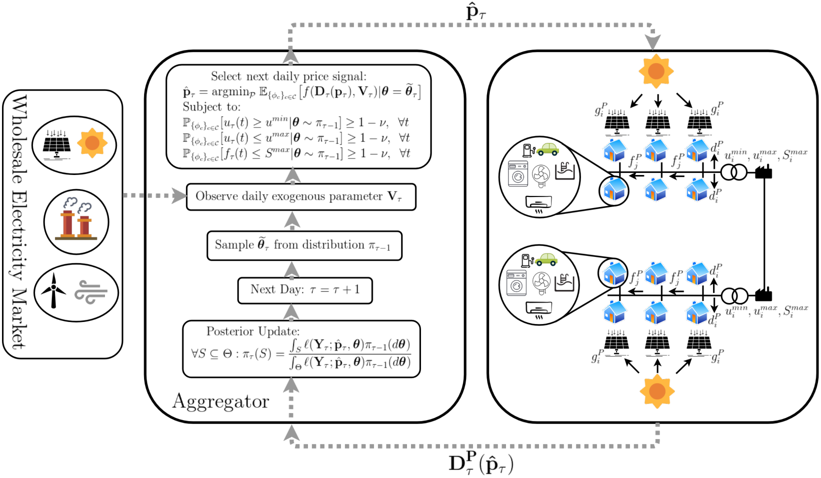
[1]: A. Gopalan, S. Mannor, Y. Mansour, 2014
[2]: A. Moradipari, C. Silva, M. Alizadeh, 2018

Con-TS-RTP



$\hat{\mathbf{p}}_\tau$
Wholesale Electricity Market
Select next daily price signal:
$\hat{\mathbf{p}}_\tau = \operatorname{argmin}_{\mathcal{P}} \mathbb{E}_{\{\phi_c\}_{c\in\mathcal{C}}}[f(\mathbf{D}_\tau(\mathbf{p}_\tau), \mathbf{V}_\tau) | \boldsymbol{\theta} = \widetilde{\boldsymbol{\theta}}_\tau]$
Subject to:
$\mathbb{P}_{\{\phi_c\}_{c\in\mathcal{C}}}[u_\tau(t) \geq u^{min} | \boldsymbol{\theta} = \widetilde{\boldsymbol{\theta}}_\tau] \geq 1 - \mu, \quad \forall t$
$\mathbb{P}_{\{\phi_c\}_{c\in\mathcal{C}}}[u_\tau(t) \leq u^{max} | \boldsymbol{\theta} = \widetilde{\boldsymbol{\theta}}_\tau] \geq 1 - \mu, \quad \forall t$
$\mathbb{P}_{\{\phi_c\}_{c\in\mathcal{C}}}[f_\tau(t) \leq S^{max} | \boldsymbol{\theta} = \widetilde{\boldsymbol{\theta}}_\tau] \geq 1 - \mu, \quad \forall t$
Observe daily exogenous parameter $\mathbf{V}_\tau$
Sample $\widetilde{\boldsymbol{\theta}}_\tau$ from distribution $\pi_{\tau-1}$
Next Day: $\tau = \tau + 1$
Posterior Update:
$\forall S \subseteq \Theta : \pi_\tau(S) = \frac{\int_S \ell(\mathbf{Y}_\tau; \hat{\mathbf{p}}_\tau, \boldsymbol{\theta}) \pi_{\tau-1}(d\boldsymbol{\theta})}{\int_\Theta \ell(\mathbf{Y}_\tau; \hat{\mathbf{p}}_\tau, \boldsymbol{\theta}) \pi_{\tau-1}(d\boldsymbol{\theta})}$
Aggregator
g_i^P
f_j^P
d_i^P
$u_i^{min}, u_i^{max}, S_i^{max}$
$\mathbf{D}_\tau^{\mathbf{P}}(\hat{\mathbf{p}}_\tau)$

Con-TS-RTP with Modified Reliability Constraints



Reliability of Con-TS-RTP

- Assumption 4: $\mathsf{KL}\Big[\ell(\mathbf{D}(\mathbf{p}); \mathbf{p}, \boldsymbol{\theta}^\star), \ell(\mathbf{D}(\mathbf{p}); \mathbf{p}, \boldsymbol{\theta})\Big] \geq \xi^\star$

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- The true parameter's ($\boldsymbol{\theta}^\star$) load profile is separable from other candidate parameters' load profiles ($\boldsymbol{\theta} \neq \boldsymbol{\theta}^\star$)

Under assumptions 1-4, the Con-TS-RTP algorithm with modified reliability constraints will uphold the distribution grid operational constraints with probability at least $1 - u$ each day.

Simple Comparison

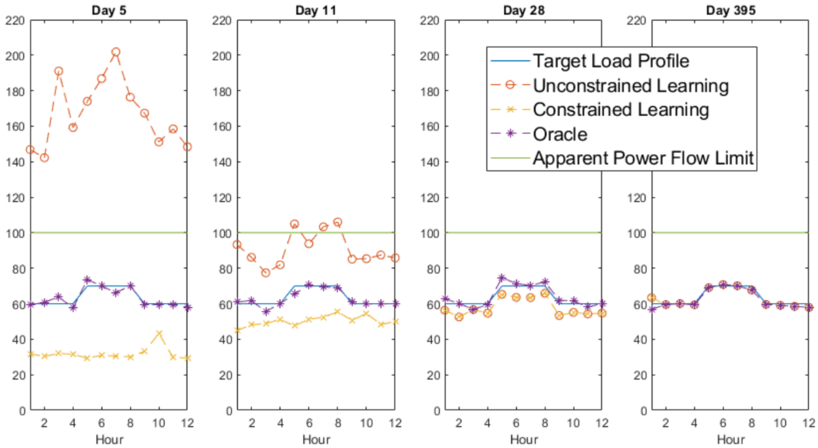

Figure: Unconstrained vs constrained Thompson Sampling for load shaping with a maximum power constraint

Radial Distribution System Test Case

Learning the True Parameter

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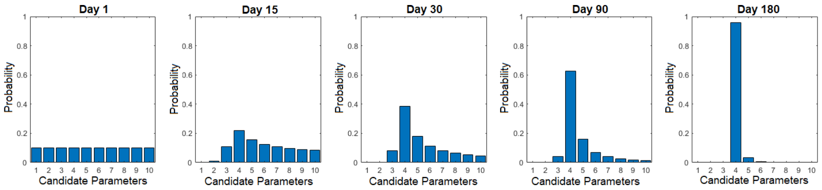
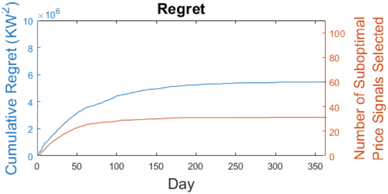


Figure: Evolution of the aggregator's knowledge of the true parameter.

Radial Distribution System Test Case

Performance



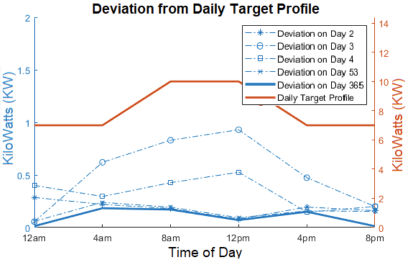


Figure: Left: Regret at node 10 with $\nu = 0.1$. Right: Deviation of node 10's demand from a specific daily target profile.

Radial Distribution System Test Case

Performance

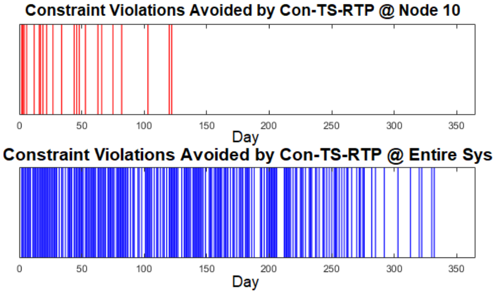


Figure: Distribution system constraint violations avoided by using Con-TS-RTP instead of an unconstrained TS.

Radial Distribution System Test Case

Performance

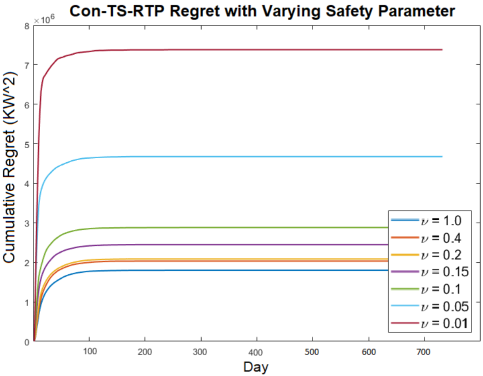


Figure: Regret curves for various system reliability metrics. Each curve is an average of 20 independent simulations.

Conclusion

Con-TS-RTP: an online learning and pricing strategy based on Thompson Sampling for an electricity aggregator attempting to learn customers' electricity usage models while implementing a load shaping program via real-time dispatch signals.

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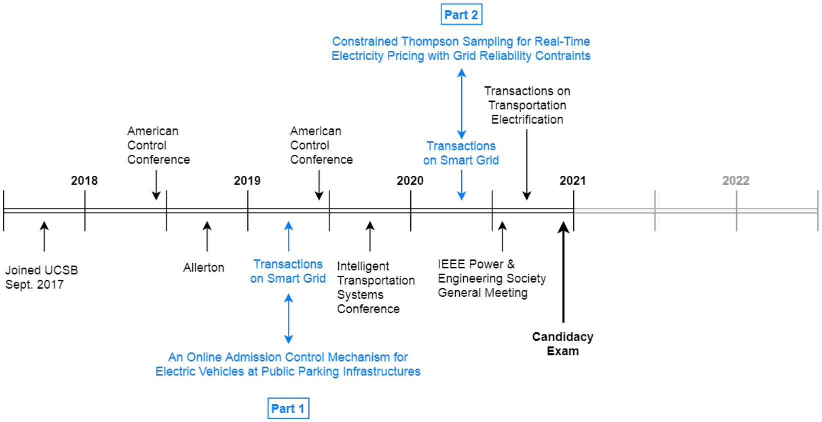
Furthermore, Con-TS-RTP accounts for the operation constraints of a distribution system to ensure adequate service and to avoid potential grid failures.

Future Plans

Timeline



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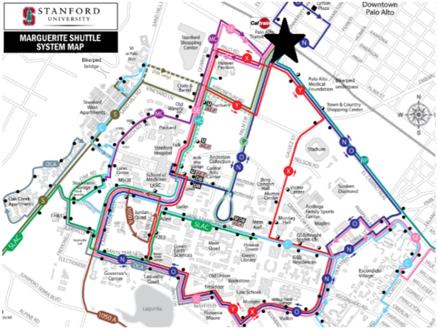
- *Virtual Shared Energy Storage* would require a scheduling and pricing mechanism for charging, discharging, and capacity
- Charging and discharging profiles *cancel* each other
- Incentivize diverse usage patterns to enable charge/discharge cancellations

Thank you!

- Mahnoosh Alizadeh
- Committee
- Gustavo Cezar
- Smart Infrastructure Systems Lab
- UCSB ECE graduate students

Other Work

Stanford Marguerite Shuttle



Route Name	Daily Trips	Trip Miles
C Line	33	7.00
C Limited	11	4.60
MC Line (AM/PM)	46	3.00
MC Line (Mid Day)	11	5.10
P Line (AM/PM)	56	2.50
P Line (Mid Day)	11	4.00
Research Park (AM/PM)	24	10.40
X Express (AM)	12	1.20
X Line	44	4.60
X Limited (AM)	10	2.00
X Limited (PM)	10	1.50
Y Express (PM)	20	1.20
Y Line	44	4.60
Y Limited (AM)	10	2.40
Y Limited (PM)	10	2.00
Totals	**352 trips/day**	**1431.50 miles/day**

Figure: Left: Primary service area for Stanford University's Marguerite Shuttle. Right: Stanford Marguerite Shuttle Route Information

Other Work

SLAC & Google Workplace Smart Charging

- Goal: Implement EV load shifting to minimize electricity cost and to ensure total EV charging load does not exceed transformer capacity
- Utilizing scenario generation and stochastic programming to schedule EV charging
- Currently working on implementing algorithm at a SLAC test site and then a Google parking lot

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- The algorithm then makes an observation dependent on the selected price and the hidden parameter and updates the parameter's distribution π_τ based on the new observation

Performance Evaluation: *Regret*

(Pseudo) Regret:

$$R_{\mathcal{T}} = \mathbb{E}\left[\sum_{\tau=1}^{\mathcal{T}} f(\mathbf{D}_\tau(\mathbf{p}_\tau), \mathbf{V}_\tau) - \sum_{\tau=1}^{\mathcal{T}} f(\mathbf{D}_\tau(\mathbf{p}^\star), \mathbf{V}_\tau)\right]$$

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Alternative:

$$\sum_{\mathbf{V}\in\mathcal{V}} \sum_{\mathbf{p}\in\{\mathcal{P}\setminus\mathbf{p}^{\mathbf{V},\star}\}} N_{\mathcal{T}}(\mathbf{p}, \mathbf{V}) = \sum_{\tau=1}^{\mathcal{T}} \mathbb{1}^{\{\mathbf{p}_\tau \neq \mathbf{p}^{\mathbf{V}_\tau,\star}\}}$$

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- Assumption 4: The true parameter's ($\boldsymbol{\theta}^\star$) load profile can be sufficiently distinguished from other candidate parameters' load profiles ($\boldsymbol{\theta} \neq \boldsymbol{\theta}^\star$)

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- Under assumptions 1-4, Gopalan, et al. [1] showed that the mass of the true parameter will not decrease below a certain threshold
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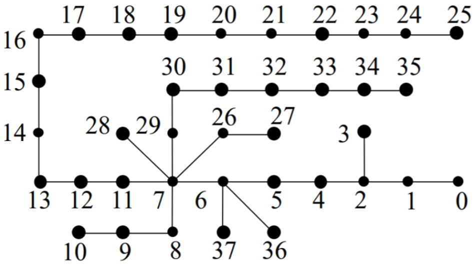
  - $\pi_\tau(\boldsymbol{\theta}^\star) \geq \pi_{min}^{\xi^\star} \quad \forall \tau$
- With ν chosen such that $\nu \leq \mu \pi_{min}^{\xi^\star}$, the total mass of the incorrect parameters ($\boldsymbol{\theta} \neq \boldsymbol{\theta}^\star$) in the prior π_τ can never be large enough to satisfy the constraint's inequality without the true parameter also satisfying the constraint

Experimental Evaluation



Line	R $(10^{-3}\Omega)$	X $(10^{-3}\Omega)$	S^{max} (KVA)	Line	R $(10^{-3}\Omega)$	X $(10^{-3}\Omega)$	S^{max} (KVA)
1	24.2	48.2	54	20	129.5	30.9	10.8
2	227.3	743.5	84	21	15.1	5.4	14.4
3	76.3	18.2	10.8	22	50.8	12.1	10.8
4	43.6	142.7	84	23	69.1	16.5	10.8
5	25.8	84.4	84	24	31.6	11.2	14.4
6	10.5	10.7	40.2	25	96.3	23	10.8
7	23.2	23.6	40.2	26	110.7	112.6	40.2
8	75.1	26.7	14.4	27	2.1	0.7	14.4
9	114.4	27.3	10.8	28	242.1	86.2	14.4
10	110.8.3	67.7	14.4	29	27.3	27.8	40.2
11	63.7	22.7	14.4	30	174.6	62.1	16.2
12	278.7	99.2	14.4	31	43	15.3	10.8
13	254.2	10.8.5	14.4	32	207.8	74	10.8
14	21.8	5.2	10.8	33	109.4	38.9	14.4
15	57.3	20.4	14.4	34	50.5	18	14.4
16	126.7	45.1	14.4	35	165.2	58.8	14.4
17	48.6	11.6	10.8	36	49.5	17.6	14.4
18	95.1	22.7	10.8	37	5.8	2.1	14.4
19	137.3	32.8	10.8				

Figure: Radial Distribution System and Parameters

LinDistFlow Equations

$$d^P_{i,\tau}(t) + \sum_{j \in \mathcal{K}_i} f^P_{j,\tau}(t) = f^P_{\mathcal{A}_i,\tau}(t);\ \ \forall t, \tau, i$$

$$d^Q_{i,\tau}(t) + \sum_{j \in \mathcal{K}_i} f^Q_{j,\tau}(t) = f^Q_{\mathcal{A}_i,\tau}(t);\ \ \forall t, \tau, i$$

$$u_{\mathcal{A}_i,\tau}(t) - 2\big(f^P_{i,\tau}(t)R_i + f^Q_{i,\tau}(t)X_i\big) = u_{i,\tau}(t); \forall t, \tau, i$$